A VARIATIONAL EQUILIBRIUM FORMULATION FOR HUMANITARIAN ORGANIZATIONS UNDER COMPETITION

Patrizia DANIELE
University of Catania - Italy

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Outline

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Disasters

Disasters have a catastrophic effect on human lives and a region's or even a nation's resources. A total of 2.3 billion people were affected by natural disasters from 1995-2015 (UN Office of Disaster Risk (2015)).

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The number of disasters is growing as well as the number of people affected by them with additional pressures coming from:

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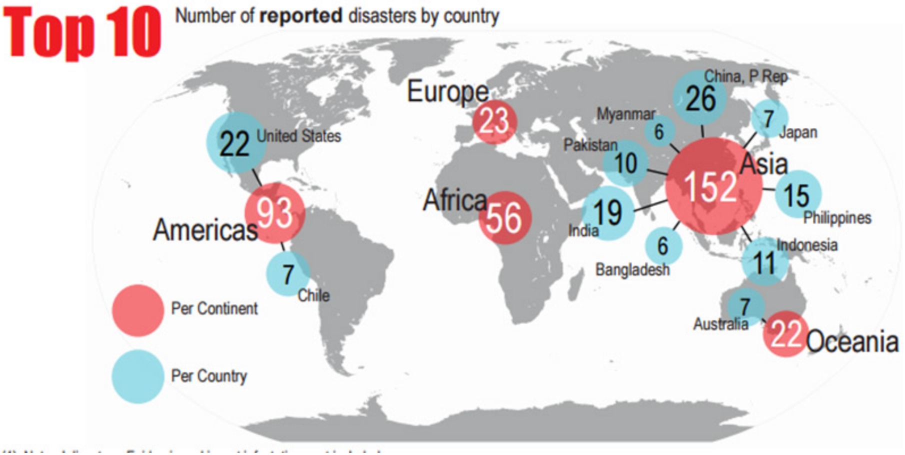
Costs

The associated costs of the damage and losses due to natural disasters is estimated at an average $117 billion a year between 1991 and 2010 (Watson et al. (2015))

Some recent disasters in the world (2015)



Hurricane Katrina in 2005

Hurricane Katrina has been called an "American tragedy", in which essential services failed completely.

The Triple Disaster (earthquake, tsunami, meltdown) in Japan on March 11, 2011

More than 18,000 people died on March 11, 2011 after the strongest recorded earthquake in Japan's history triggered a tsunami that laid waste to entire towns and villages and caused a triple meltdown at the Fukushima Daiichi nuclear power plant

The Superstorm Sandy 2012

Superstorm Sandy was the deadliest and most destructive hurricane of the 2012 Atlantic hurricane season

Earthquake in Nepal 2015

The April 2015 Nepal earthquake killed nearly 9,000 people and injured nearly 22,000, with maximum Mercalli Intensity of VIII (Severe)

Hurricane Matthew in 2016

Hurricane Matthew was a tropical storm which caused catastrophic damage and a humanitarian crisis in Haiti, as well as widespread devastation in the southeastern United States

Challenges Associated with Disaster Relief

- Timely delivery of relief items is challenged by damaged and destroyed infrastructure (transportation, telecommunications, hospitals, etc.)
- Shipments of the wrong supplies create congestion and material convergence (sometimes referred to as the second disaster)

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We believe that some of the challenges that humanitarian organizations engaged in disaster relief are faced with can be addressed through the use of game theory

It is a methodological framework that captures complex interactions among competing decision-makers (noncooperative games) or cooperating ones (cooperative games)

Game Theory and Disaster Relief

We construct a new Generalized Nash Equilibrium (GNE) network model for disaster relief, where the utility function that each NGO seeks to maximize depends on its financial gain from donations plus the weighted benefit accrued from doing good through the delivery of relief items minus the total cost associated with the logistics of delivering the relief items

In our model:

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In our model:

- The financial funds function need not take on a particular structure
- The altruism or benefit functions need not be linear
- The competition associated with logistics is captured through total cost functions that depend not only on a particular NGO's relief item shipments but also on those of the other NGOs

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We make use of a Variational Equilibrium and, hence, we do not need to utilize quasi-variational inequalities in the formulation and computations but can apply the more advanced variational inequality theory.

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- m **humanitarian organizations**, here referred to as nongovernmental organizations (NGOs), with a typical NGO denoted by i
- n **demand points**, with a typical one denoted by j

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- q_{ij} : the flow of the relief item shipment (water, food, or medicine) delivered by NGO i to demand point $j \Longrightarrow q_i \in \mathbb{R}^n_+$ (*vector of strategies* of NGO i) $\Longrightarrow q \in \mathbb{R}^{mn}_+$

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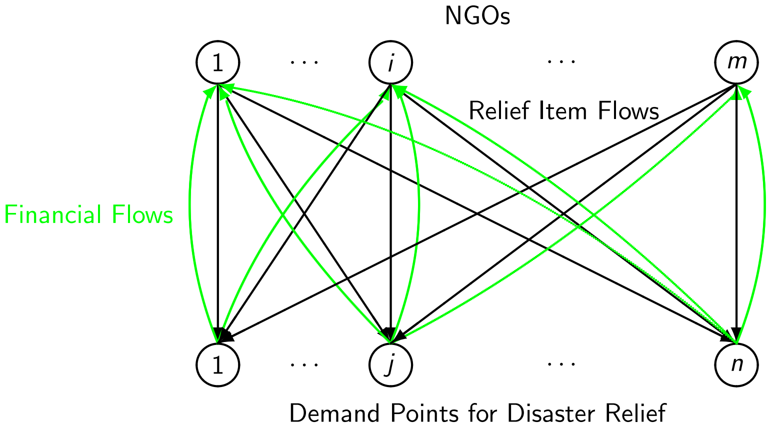
- $c_{ij}(q)$: the cost associated with shipping the relief items to location j

The Network Structure of the Model



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- $B_i(q)$: the altruism/benefit function
- ω_i : the monetized weight associated with altruism of i
- s_i : the amount of the relief item that the ONG i can allocate post-disaster

The Mathematical Model

Without the imposition of demand bound constraints:

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Optimization Problem

$$\text{Maximize} \quad U_i(q) = \sum_{j=1}^{n} P_{ij}(q) + \omega_i B_i(q) - \sum_{j=1}^{n} c_{ij}(q)$$

subject to constraints

$$\begin{aligned} \sum_{j=1}^{n} q_{ij} &\leq s_i \\ q_{ij} &\geq 0, \quad j = 1, \ldots, n \end{aligned}$$

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It is a Nash Equilibrium problem, which can be formulated as a variational inequality problem

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However, the NGOs may be faced with several common constraints, which make the game theory problem more complex and challenging.

The common constraints, which are imposed by an authority, ensure that the needs of the disaster victims are met, while recognizing the negative effects of waste and material convergence.

The Mathematical Model

The two sets of common constraints at each demand point j; $j = 1, \ldots, n$, are:

Common Constraints

$$\sum_{i=1}^{m} q_{ij} \geq \underline{d}_j$$

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We assume that:

$$\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} \underline{d}_j$$

The Mathematical Model

Feasible Set

$$K_i \equiv \left\{ q_i | \sum_{j=1}^{n} q_{ij} \leq s_i, \quad q_{ij} \geq 0, \quad j = 1, \ldots, n \right\}$$

and

$$K \equiv \prod_{i=1}^{m} K_i$$

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Feasible set of the shared constraints

$$\mathcal{S} \equiv \left\{ q \middle| \sum_{i=1}^{m} q_{ij} \geq \underline{d}_j, \quad \sum_{i=1}^{m} q_{ij} \leq \bar{d}_j, \quad \forall j = 1, \ldots, n \right\}$$

The Mathematical Model

Definition 1: Disaster Relief Generalized Nash Equilibrium

A relief item flow pattern $q^* \in K = \prod_{i=1}^{m} K_i$, $q^* \in \mathcal{S}$, constitutes a disaster relief Generalized Nash Equilibrium if for each NGO i; $i = 1, \ldots, m$:

$$U_i(q_i^*, \hat{q}_i^*) \geq U_i(q_i, \hat{q}_i^*), \quad \forall q_i \in K_i, \forall q \in \mathcal{S},$$

where $\hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*)$

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Hence, an equilibrium is established if no NGO can unilaterally improve upon its utility by changing its relief item flows in the disaster relief network, given the relief item flow decisions of the other NGOs, and subject to the supply constraints, the nonnegativity constraints, and the shared/coupling constraints. We remark that both K and $\mathcal{S}$ are convex sets.

Variational Formulation

Definition 2: Variational Equilibrium

A strategy vector q^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $q^* \in K, q^* \in \mathcal{S}$ is a solution of the variational inequality:

$$-\sum_{i=1}^{m} \langle \nabla_{q_i} U_i(q^*), q_i - q_i^* \rangle \geq 0, \quad \forall q \in K, \forall q \in \mathcal{S}. \tag{1}$$

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We have that (1) is equivalent to the variational inequality:

Variational Inequality

Find $q^* \in K, q^* \in \mathcal{S}$ such that :

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left[\sum_{k=1}^{n} \frac{\partial c_{ik}(q^*)}{\partial q_{ij}} - \sum_{k=1}^{n} \frac{\partial P_{ik}(q^*)}{\partial q_{ij}} - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}} \right] \times \left[q_{ij} - q_{ij}^* \right] \geq 0, (2)$$

$\forall q \in K, \forall q \in \mathcal{S}.$

Standard Form

Find $X^* \in \mathcal{K} \subset R^N$:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $X \equiv q$ and $F(X)$ where component (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, of $F(X)$, $F_{ij}(X)$, is given by

$$F_{ij}(X) \equiv \left[\sum_{k=1}^{n} \frac{\partial c_{ik}(q)}{\partial q_{ij}} - \sum_{k=1}^{n} \frac{\partial P_{ik}(q)}{\partial q_{ij}} - \omega_i \frac{\partial B_i(q)}{\partial q_{ij}} \right] \tag{17}$$

and $\mathcal{K} \equiv K \cap \mathcal{S}$

Lagrange Theory and Analysis of Marginal Utilities

For an application of Lagrange theory to other models, see: Daniele (2001) (spatial economic models), Barbagallo, Daniele, and Maugeri (2012) (financial networks), Toyasaki, Daniele, and Wakolbinger (2014) (end-of-life products networks), Daniele and Giuffrè (2015) (random traffic networks), Caruso and Daniele (2016) (transplant networks), Nagurney and Dutta (2016) (competition for blood donations).

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By setting:

$$C(q) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[\sum_{k=1}^{n} \frac{\partial c_{ik}(q^*)}{\partial q_{ij}} - \sum_{k=1}^{n} \frac{\partial P_{ik}(q^*)}{\partial q_{ij}} - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}} \right] (q_{ij} - q_{ij}^*),$$

variational inequality (2) can be rewritten as a minimization problem as follows:

$$\min_{\mathcal{K}} C(q) = C(q^*) = 0,$$

where all the involved functions are continuously differentiable and convex

Lagrange Theory and Analysis of Marginal Utilities

We set:

$$\begin{aligned} a_{ij} &= -q_{ij} \leq 0, & \forall i,\ \forall j, \\ b_i &= \sum_{j=1}^{n} q_{ij} - s_i \leq 0, & \forall i, \\ c_j &= \underline{d}_j - \sum_{i=1}^{m} q_{ij} \leq 0, & \forall j, \\ e_j &= \sum_{i=1}^{m} q_{ij} - \overline{d}_j \leq 0, & \forall j, \end{aligned}$$

and

$$\Gamma(q) = (a_{ij}, b_i, c_j, e_j)_{i=1,\ldots,m;\, j=1,\ldots,n}$$

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then

$$\mathcal{K} = \{q \in R^{mn} : \Gamma(q) \le 0\}$$

Lagrange Theory and Analysis of Marginal Utilities

Lagrange Function

$$\begin{aligned}\mathcal{L}(q,\alpha,\delta,\sigma,\varepsilon) &= \sum_{j=1}^{n} c_{ij}(q) - \sum_{j=1}^{n} P_{ij}(q) - \omega_i B_i(q) \\ &+ \sum_{i=1}^{m}\sum_{j=1}^{n} \alpha_{ij} a_{ij} + \sum_{i=1}^{m} \delta_i b_i + \sum_{j=1}^{n} \sigma_j c_j + \sum_{j=1}^{n} \varepsilon_j e_j,\end{aligned}$$

$$\forall q \in R_+^{mn},\ \forall \alpha \in R_+^{mn},\ \forall \delta \in R_+^{m},\ \forall \sigma \in R_+^{n},\ \forall \varepsilon \in R_+^{n},$$

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Lagrange Function

$$\mathcal{L}(q,\alpha,\delta,\sigma,\varepsilon) = \sum_{j=1}^{n} c_{ij}(q) - \sum_{j=1}^{n} P_{ij}(q) - \omega_i B_i(q) + \sum_{i=1}^{m}\sum_{j=1}^{n} \alpha_{ij} a_{ij} + \sum_{i=1}^{m} \delta_i b_i + \sum_{j=1}^{n} \sigma_j c_j + \sum_{j=1}^{n} \varepsilon_j e_j,$$

$$\forall q \in R_+^{mn},\ \forall \alpha \in R_+^{mn},\ \forall \delta \in R_+^{m},\ \forall \sigma \in R_+^{n},\ \forall \varepsilon \in R_+^{n},$$

It is easy to prove that the feasible set $\mathcal{K}$ is convex and that the Slater condition is satisfied

Lagrange Theory and Analysis of Marginal Utilities

Then, if q^* is a minimal solution, there exist $\alpha^* \in R_+^{mn}$, $\delta^* \in R_+^m$, $\sigma^* \in R_+^n$, $\varepsilon^* \in R_+^n$ such that the vector $(q^*, \alpha^*, \delta^*, \sigma^*, \varepsilon^*)$ is a saddle point of the Lagrange function; namely:

$$\mathcal{L}(q^*, \alpha, \delta, \sigma, \varepsilon) \leq \mathcal{L}(q^*, \alpha^*, \delta^*, \sigma^*, \varepsilon^*) \leq \mathcal{L}(q, \alpha^*, \delta^*, \sigma^*, \varepsilon^*),$$

$$\forall q \in R_+^{mn},\ \forall \alpha \in R_+^{mn},\ \forall \delta \in R_+^m,\ \forall \sigma \in R_+^n,\ \forall \varepsilon \in R_+^n,$$

and

$$\alpha_{ij}^* a_{ij}^* = 0,\ \forall i,\ \forall j,$$

$$\delta_i^* b_i^* = 0,\ \forall i,$$

$$\sigma_j^* c_j^* = 0, \quad \varepsilon_j^* e_j^* = 0,\ \forall j.$$

Lagrange Theory and Analysis of Marginal Utilities

From the right-hand side, it follows that $q^* \in R_+^{mn}$ is a minimal point of $\mathcal{L}(q, \alpha^*, \delta^*, \sigma^*, \varepsilon^*)$ in the whole space R^{mn}, and hence, for all $i = 1, \ldots, m$, and for all $j = 1, \ldots, n$, we have that:

$$\frac{\partial \mathcal{L}(q^*, \alpha^*, \delta^*, \sigma^*, \varepsilon^*)}{\partial q_{ij}}$$

$$= \sum_{k=1}^{n} \frac{\partial c_{ik}(q^*)}{\partial q_{ij}} - \sum_{k=1}^{n} \frac{\partial P_{ik}(q^*)}{\partial q_{ij}} - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}} - \alpha_{ij}^* + \delta_i^* - \sigma_j^* + \varepsilon_j^* = 0$$

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Hence:

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Lagrange Theory and Analysis of Marginal Utilities

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} \underbrace{\alpha_{ij}^{*} q_{ij}}_{\geq 0} - \sum_{i=1}^{m} \delta_i^{*} \left(\underbrace{\sum_{j=1}^{n} q_{ij} - s_i}_{\leq 0} \right) + \sum_{j=1}^{n} \sigma_j^{*} \left(\underbrace{\sum_{i=1}^{m} q_{ij} - \underline{d}_j}_{\geq 0} \right)$$

$$- \sum_{j=1}^{n} \varepsilon_j^{*} \left(\underbrace{\sum_{i=1}^{m} q_{ij} - \overline{d}_j}_{\leq 0} \right) \geq 0$$

Interpretation of the Lagrange Multipliers

We now discuss the meaning of some of the Lagrange multipliers. We focus on the case where $q^*_{ij} > 0$; namely, the relief item flow from NGO i to demand point j is positive; otherwise, if $q^*_{ij} = 0$, the problem is not interesting.

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Let us consider the situation when the constraints are not active, that is, $b^*_i < 0$ and $\underline{d}_j < \sum_{i=1}^{m} q^*_{ij} < \overline{d}_j$.

Specifically, $b^*_i < 0$ means that $\sum_{j=1}^{n} q^*_{ij} < s_i$; that is, the sum of relief items sent by the i-th NGO to all demand points is strictly less than the total amount s_i at its disposal. Then, we get: $\delta^*_i = 0$.

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Hence:

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$$\iff \sum_{k=1}^{n} \frac{\partial P_{ik}(q^*)}{\partial q_{ij}} + \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}} = \sum_{k=1}^{n} \frac{\partial c_{ik}(q^*)}{\partial q_{ij}}$$

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In this case, the marginal utility associated with the financial donations plus altruism is equal to the marginal costs.

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Hence:

$$\sum_{k=1}^{n} \frac{\partial c_{ik}(q^*)}{\partial q_{ij}} - \sum_{k=1}^{n} \frac{\partial P_{ik}(q^*)}{\partial q_{ij}} - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}} = \alpha_{ij}^* - \delta_i^* + \sigma_j^* - \varepsilon_j^* = 0$$

$$\Longleftrightarrow \sum_{k=1}^{n} \frac{\partial P_{ik}(q^*)}{\partial q_{ij}} + \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}} = \sum_{k=1}^{n} \frac{\partial c_{ik}(q^*)}{\partial q_{ij}}$$

In this case, the marginal utility associated with the financial donations plus altruism is equal to the marginal costs.

If, on the other hand, $\sum_{i=1}^{m} q_{ij}^* = \underline{d}_j$, then $\sigma_j^* > 0$. Hence, we get:

$$\sum_{k=1}^{n} \frac{\partial P_{ik}(q^*)}{\partial q_{ij}} + \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}} + \sigma_j^* = \sum_{k=1}^{n} \frac{\partial c_{ik}(q^*)}{\partial q_{ij}}, \text{ with } \sigma_j^* > 0,$$

Interpretation of the Lagrange Multipliers

and, therefore,

$$\sum_{k=1}^{n} \frac{\partial c_{ik}(q^*)}{\partial q_{ij}} > \sum_{k=1}^{n} \frac{\partial P_{ik}(q^*)}{\partial q_{ij}} + \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}},$$

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Finally, if $\sum_{i=1}^{m} q_{ij}^* = \overline{d}_j$, then $\varepsilon_j^* > 0$, we have that:

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Therefore,

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From the above analysis of the Lagrange multipliers and marginal utilities at the equilibrium solution, we can conclude that the most convenient situation, in terms of the marginal utilities, is the one when $\sum_{i=1}^{m} q_{ij}^* = \overline{d}_j$ and $\sum_{j=1}^{n} q_{ij}^* = s_i$.

Equivalent Variational Formulation

$$\text{Find } (q^*, \delta^*, \sigma^*, \varepsilon^*) \in R_+^{mn+m+2n} :$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left[\sum_{k=1}^{n} \frac{\partial c_{ik}(q^*)}{\partial q_{ij}} - \sum_{k=1}^{n} \frac{\partial P_{ik}(q^*)}{\partial q_{ij}} - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}} + \delta_i^* - \sigma_j^* + \varepsilon_j^* \right] (q_{ij} - q_{ij}^*)$$

$$+ \sum_{i=1}^{m} \left(s_i - \sum_{j=1}^{n} q_{ij}^* \right) (\delta_i - \delta_i^*)$$

$$+ \sum_{j=1}^{n} \left(\sum_{i=1}^{m} q_{ij}^* - \underline{d}_j \right) (\sigma_j - \sigma_j^*) + \sum_{j=1}^{n} \left(\overline{d}_j - \sum_{i=1}^{m} q_{ij}^* \right) (\varepsilon_j - \varepsilon_j^*) \geq 0,$$

$$\forall q \in R_+^{mn}, \ \forall \delta \in R_+^m, \ \forall \sigma \in R_+^n, \ \forall \varepsilon \in R_+^n.$$

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Our case study is inspired by a disaster consisting of a series of tornados that hit western Massachusetts on June 1, 2011 in the late afternoon.

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- 319 destroyed
- 593 sustaining major damage
- 273 sustaining minor damage
- 250 otherwise affected

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FEMA estimated that the primary impact was damage to buildings and equipment with a cost estimate of $24,782,299. Total damage estimates from the storm exceeded $140 million, the majority from the destruction of homes and businesses

Numerical Examples

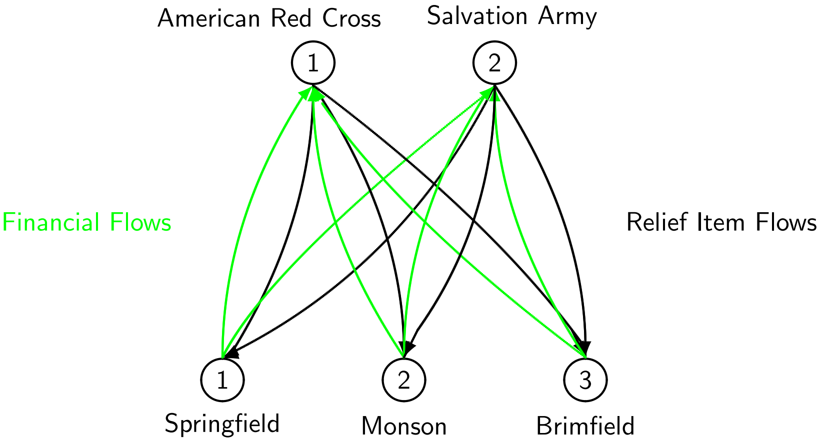


Figure: The Network Topology for the Case Study, Example 1

Numerical Examples

Supplies of Meals and Weights

$$s_1 = 25,000, \quad s_2 = 25,000$$
$$\omega_1 = 1, \quad \omega_2 = 1$$

Financial Funds Functions

$$P_{11}(q) = 1000\sqrt{(3q_{11} + q_{21})}, \quad P_{12}(q) = 600\sqrt{(2q_{12} + q_{22})}$$
$$P_{13}(q) = 400\sqrt{(2q_{13} + q_{23})}, \quad P_{21}(q) = 800\sqrt{(4q_{21} + q_{11})}$$
$$P_{22}(q) = 400\sqrt{(2q_{22} + q_{12})}, \quad P_{23}(q) = 200\sqrt{(2q_{23} + q_{13})}$$

Altruism Functions

$$B_1(q) = 300q_{11} + 200q_{12} + 100q_{13}, \quad B_2(q) = 400q_{21} + 300q_{22} + 200q_{23}$$

Numerical Examples

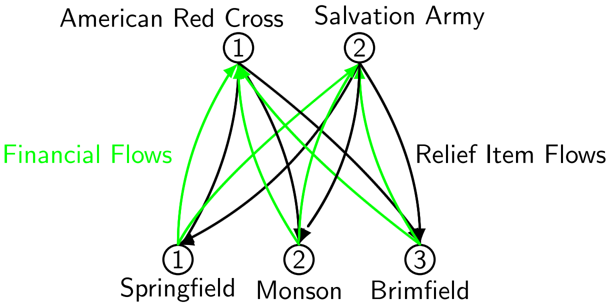
Cost Functions

$$c_{11}(q) = .15q_{11}^2 + 2q_{11}, \quad c_{12}(q) = .15q_{12}^2 + 5q_{12}, \quad c_{13}(q) = .15q_{13}^2 + 7q_{13}$$
$$c_{21}(q) = .1q_{21}^2 + 2q_{21}, \quad c_{22}(q) = .1q_{22}^2 + 5q_{22}, \quad c_{23}(q) = .1q_{23}^2 + 7q_{23}$$

Demand Lower and Upper Bounds

$$\underline{d}_1 = 10000, \quad \bar{d}_1 = 20000$$
$$\underline{d}_2 = 1000, \quad \bar{d}_2 = 10000$$
$$\underline{d}_3 = 1000, \quad \bar{d}_3 = 10000$$

Optimal Solutions



$$q_{11}^* = 3800.24, \quad q_{12}^* = 668.64, \quad q_{13}^* = 326.66,$$

$$q_{21}^* = 6199.59, \quad q_{22}^* = 1490.52, \quad q_{23}^* = 974.97.$$

$$\sum_{j=1}^{3} P_{1j}(q^*) = 180,713.23, \quad \sum_{j=1}^{3} P_{2j}(q^*) = 168,996.78.$$

Numerical Examples

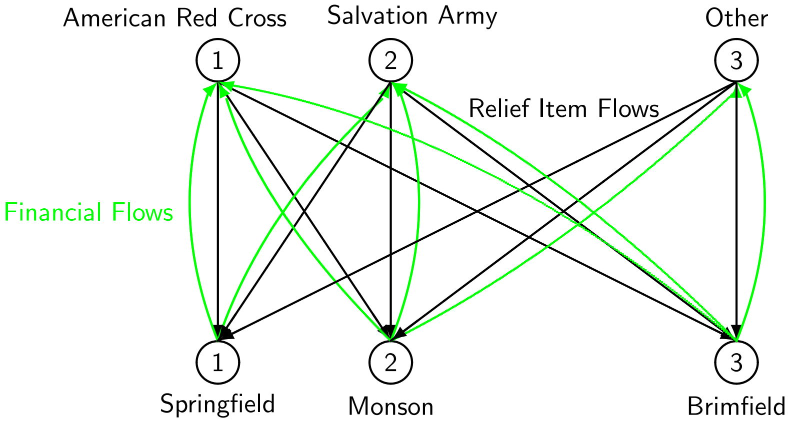


Figure: The Network Topology for the Case Study, Example 2

Numerical Examples

The unspecified data are as in Example 1.

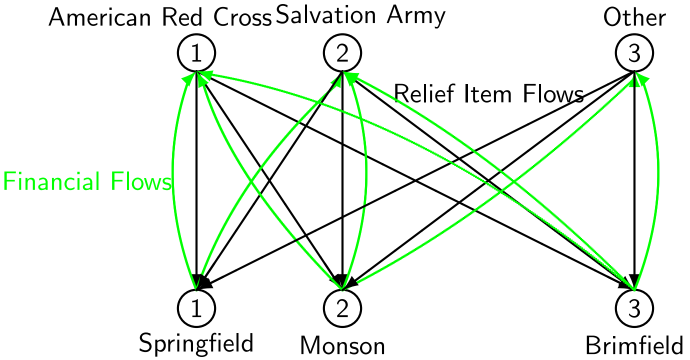
Financial Funds Functions

$$P_{11}(q) = 1000\sqrt{(3q_{11} + q_{21} + q_{31})}, \quad P_{12}(q) = 600\sqrt{(2q_{12} + q_{22} + q_{32})},$$
$$P_{13}(q) = 400\sqrt{(2q_{13} + q_{23} + q_{33})}, \quad P_{21}(q) = 800\sqrt{(4q_{21} + q_{11} + q_{31})},$$
$$P_{22}(q) = 400\sqrt{(2q_{22} + q_{12} + q_{32})}, \quad P_{23}(q) = 200\sqrt{(2q_{23} + q_{13} + q_{33})},$$
$$P_{31}(q) = 400\sqrt{(2q_{31} + q_{11} + q_{21})}, \quad P_{32}(q) = 200\sqrt{(2q_{32} + q_{12} + q_{22})},$$
$$P_{33}(q) = 100\sqrt{(2q_{33} + q_{13} + q_{23})}.$$

Weight. Altruism/Benefit Function. Cost Functions

$$\omega_3 = 1 \quad B_3(q) = 200q_{31} + 100q_{32} + 100q_{33}$$
$$c_{31}(q) = .1q_{31}^2 + q_{31}, \quad c_{32}(q) = .2q_{32}^2 + 5q_{32}, \quad c_{33}(q) = .2q_{33}^2 + 7q_{33}$$

Optimal Solutions



$$q^*_{11} = 2506.97, \quad q^*_{12} = 667.85, \quad q^*_{13} = 325.59,$$
$$q^*_{21} = 4259.59, \quad q^*_{22} = 1489.98, \quad q^*_{23} = 974.45,$$
$$q^*_{31} = 3233.35, \quad q^*_{32} = 242.42, \quad q^*_{33} = 235.52.$$

$$\sum_{j=1}^{3} P_{1j}(q^*) = 173,021.70, \quad \sum_{j=1}^{3} P_{2j}(q^*) = 155,709.50, \quad \sum_{j=1}^{3} P_{3j}(q^*) = 60,504.14.$$

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