



INNOVATIVE STRATEGIES FOR SUSTAINABLE AND EFFICIENT LAST-MILE DELIVERY

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Ministero
dell'Università
e della Ricerca



Italiadomani
PIANO NAZIONALE
DI RIPRESA E RESILIENZA



ecosustAinable effiCient tecH-driven Last miLE logiStics



- University of Naples Federico II
- Institute of System Analysis and Computer Science “Antonio Ruberti” of the National Research Council (IASI-CNR)
- University of Catania

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E-commerce

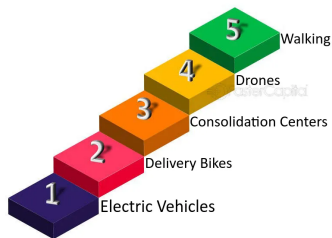
- The rapid growth of e-commerce has intensified the need for efficient last-mile delivery solutions



Environmental concerns

- Rise of **environmental concerns** and the emergence of **new business models** have prompted companies to explore alternative strategies to optimize last-mile delivery

Environmental Impact of Last-Mile Delivery



Green Vehicles

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- One promising approach is the integration of **green vehicles** into the last-mile delivery process (represent a good **alternative** to petrol and diesel-powered fleets, reduce **carbon emissions** and reduce **operational costs**)



Crowdsourcing

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- Another approach is the integration of **crowdsourced drivers** into the last-mile delivery process

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Regular Vehicles and Occasional Drivers

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- We develop a **model** to optimize last-mile delivery operations using a **combination of RVs and ODs** (individuals who can transport goods as part of their **daily routines**). Also this approach results in significant **cost reductions and increased efficiency**



Parcel Lockers

- Another approach is the use of **parcel lockers** which eliminate the inconvenience of missed deliveries and restrictive delivery schedules.



Parcel Lockers

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Advantages:

They are very useful in **urban logistics** and provide numerous advantages for consumers and companies.

For **consumers**:

- eliminate the inconvenience of **missed deliveries** and restrictive delivery schedules
- provide **24/7 accessibility**
- reduce **dependency on home delivery**

Parcel Lockers

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For **consumers**:

- eliminate the inconvenience of **missed deliveries** and restrictive delivery schedules
- provide **24/7 accessibility**
- reduce **dependency on home delivery**

For **companies**, like Amazon:

- contribute to **cost reduction**, since multiple deliveries into a single location are allowed
- minimize **failed delivery attempts**
- reduce **carbon emissions**

Truck-drone Systems

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- Another solution is the **truck-drone system** (which typically consists of **delivery trucks** which serve as **mobile hubs** from which drones are deployed to deliver packages to customers).



Truck-drone Systems

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Advantages:

- Drones can handle **short-distance deliveries** (particularly in congested urban areas or remote rural locations)
- Significantly reduce **total delivery times**
- Significantly reduce **energy consumption** compared to conventional truck-only routes.

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- 1 C. Archetti, M. Savelsbergh, M.G. Speranza, *The Vehicle Routing Problem with Occasional Drivers*, European Journal of Operational Research, 254, 2016, 472-480:
the authors examine a scenario where a company can use both its own fleet of **capacitated vehicles and drivers**, as well as **occasional drivers** who make a single delivery using their own vehicles for a **small compensation**, provided the delivery is near their destination. The company's goal is to **minimize total delivery costs**.

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- 2 C. Archetti, F. Guerriero, G. Macrina, *The online vehicle routing problem with occasional drivers*, Computers and Operations Research, 127, 2021:
the authors study a vehicle routing problem where customer **requests** are either **pre-planned** or arrive **dynamically** during distribution. The company uses a fixed fleet of vehicles, supplemented by occasional drivers. **Penalties** apply for time window violations or unserved requests. The goal is to **minimize distribution costs**.

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- 1 M. Boccia, A. Mancuso, A. Masone, C. Sterle, *Exact and heuristic approaches for the Truck-Drone Team Logistics Problem*, Transportation Research Part C: Emerging Technologies, 165, August 2024, 104691:
the authors address the **truck-and-drone routing problem**, where drones can visit multiple customers per flight. They examine scenarios where the drone either lands or hovers at recollection points.

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the authors address the **truck-and-drone routing problem**, where drones can visit multiple customers per flight. They examine scenarios where the drone either lands or hovers at recollection points.
- ➋ G. Colajanni, P. Daniele, A. Nagurney, *Centralized Supply Chain Network Optimization with UAV-based last mile deliveries*, Transportation Research Part C, 155, 2023, 104316:
we develop a **centralized supply chain network** optimization model to **maximize a company's total profit**, using a fleet of trucks and drones for last-mile delivery. We have to take into account some realistic limitations due to drones, such as low battery capacity and limited delivery range

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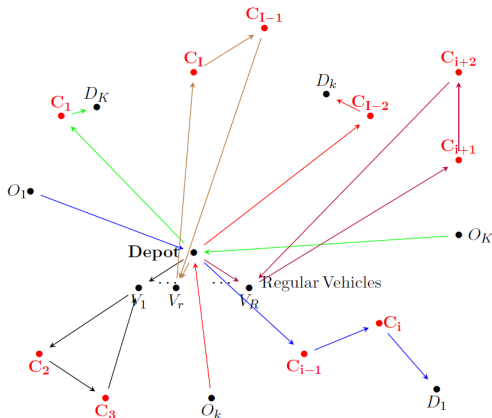


Figure: Network topology

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Assumptions:

- 1 All customers' requests have to be satisfied

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Assumptions:

- 1 All customers' requests have to be satisfied
- 2 Both ODs and RVs can serve more than one customer

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Assumptions:

- 1 All customers' requests have to be satisfied
- 2 Both ODs and RVs can serve more than one customer
- 3 Both ODs and RVs can use petrol/diesel vehicles or electric vehicles

The Supply Chain Network

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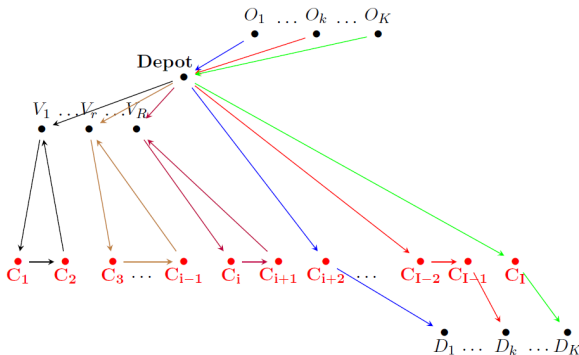


Figure: Supply chain representation

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Company's objective

$$\begin{aligned} \max \left\{ \sum_{i=1}^I \sum_{l=1}^2 p_i^l \left[\sum_{r=1}^R x_{ri}^l + \sum_{k=1}^K y_{ki}^l \right] + N \sum_{i=1}^I \left[\sum_{r=1}^R x_{ri}^2 + \sum_{k=1}^K y_{ki}^2 \right] \right. \\ - \gamma(x, y) - \sum_{r=1}^R \sum_{l=1}^2 (c_r^{Bl} + c_r^{Ml} + c_r^{Sl}(x_r^l)) - \sum_{r=1}^R c_r^D \\ - \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^2 c_{ri}^l(x_{ri}^l) - \sum_{k=1}^K c^{OD} - \sum_{k=1}^K \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) \\ \left. - \sum_{i=1}^I \sum_{l=1}^2 P_i^l(\rho_i^l) \left[\sum_{r=1}^R d_{ri}^l + \sum_{k=1}^K d_{ki}^l \right] \right\} \end{aligned}$$

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Costs

- **Management** costs
- **Amortized** costs (which include insurance, road tax, total service costs (which depend on the use of the vehicles))
- Total costs of the Regular Drivers
- **Shipping** costs for delivery (fuel/electricity charge)
- **Agreement** costs for all Occasional Drivers
- **Penalties** that company has to pay if customers experience late deliveries, regardless of whether the delay is caused by RVs or ODs.

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Constraints

- Maximum **capacity** of each RV (per vehicle) of the company:

$$\sum_{i=1}^I x_{ri}^I \leq \bar{C}_r^I \quad \forall r = 1, \dots, R, \quad \forall I = 1, 2$$

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$$\sum_{i=1}^I x_{ri}^l \leq \bar{C}_r^l \quad \forall r = 1, \dots, R, \quad \forall l = 1, 2$$

- Delay** (given by the difference between the sum of the delivery times to the customers preceding i and the time within which to make the delivery to i):

$$d_{ri}^l \geq \sum_{j=1}^I \delta_{ji}^l t_{rj}^l (x_r^l) - \bar{T}_i, \quad \forall r = 1, \dots, R, \quad \forall i = 1, \dots, I, \quad \forall l = 1, 2$$

$$d_{ri}^l \geq 0, \quad \forall r = 1, \dots, R, \quad \forall i = 1, \dots, I, \quad \forall l = 1, 2$$

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ODs' objective

$$\max \left\{ \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) - \sum_{l=1}^2 \tilde{c}_k^{Ml} - \sum_{l=1}^2 \tilde{c}_k^{Sl}(y_k^l) - \sum_{i=1}^I \sum_{l=1}^2 \tilde{c}_{ki}^l(y_{ki}^l) - \alpha_k \sum_{i=1}^I V_i d_{ki} \right\}$$

Costs:

- **amortized** costs of maintaining any type of vehicle
- **service** costs
- actual **shipping** costs to customers
- **negative evaluation** received by the OD in case of delay on the delivery to customers and we consider its economic perception

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Constraints

- Maximum **capacity** of OD k , at time t (for both types of vehicles):

$$\sum_{i=1}^I y_{ki}^I \leq \bar{c}_k^I \quad \forall I = 1, 2$$

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Constraints

- Maximum **capacity** of OD k , at time t (for both types of vehicles):

$$\sum_{i=1}^l y_{ki}^l \leq \bar{C}_k^l \quad \forall l = 1, 2$$

- Availability** of OD k to visit a customer with a vehicle of type l :

$$y_{ki}^l \leq M\beta_{ki}^l \quad \forall i = 1, \dots, l, \quad \forall l = 1, 2$$

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$$\sum_{i=1}^l y_{ki}^l \leq \overline{C}_k \quad \forall l = 1, 2$$

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$$y_{ki}^l \leq M\beta_{ki}^l \quad \forall i = 1, \dots, l, \quad \forall l = 1, 2$$

- Delay** (given by the difference between the total delivery times and the time within which delivery to i must be made):

$$d_{ki}^l \geq \sum_{j=1}^l \delta_{ji}^{kl} t_{kj}^l(y_k^l) - \overline{T}_i, \quad \forall i = 1, \dots, l, \quad \forall l = 1, 2.$$

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Constraints

- Maximum **time available** for k , with vehicle l :

$$\sum_{i=1}^l t_{ki}^l(y_k^l) \leq \bar{T}_k^l, \quad \forall l = 1, 2.$$

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Customers' equilibrium conditions

$$p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^l) d_{ri}^l \begin{cases} = \rho_i^{l*} & \text{if } x_{ri}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } x_{ri}^{l*} = 0 \end{cases} \quad \forall r = 1, \dots, R, \quad \forall l = 1, 2$$

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Customers' equilibrium conditions

$$p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^l) d_{ri}^l \begin{cases} = \rho_i^{l*} & \text{if } x_{ri}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } x_{ri}^{l*} = 0 \end{cases} \quad \forall r = 1, \dots, R, \forall l = 1, 2$$

$$p_i^l + \hat{c}_{ki}^l(y_{ki}^{l*}) - P_i^l(\rho_i^l) d_{ki}^l \begin{cases} = \rho_i^{l*} & \text{if } y_{ki}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } y_{ki}^{l*} = 0 \end{cases} \quad \forall k = 1, \dots, K, \forall l = 1, 2$$

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$$p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^l) d_{ri}^l \begin{cases} = \rho_i^{l*} & \text{if } x_{ri}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } x_{ri}^{l*} = 0 \end{cases} \quad \forall r = 1, \dots, R, \forall l = 1, 2$$

$$p_i^l + \hat{c}_{ki}^l(y_{ki}^{l*}) - P_i^l(\rho_i^l) d_{ki}^l \begin{cases} = \rho_i^{l*} & \text{if } y_{ki}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } y_{ki}^{l*} = 0 \end{cases} \quad \forall k = 1, \dots, K, \forall l = 1, 2$$

$$\phi_i^l = \sum_{r=1}^R x_{ri}^{l*} + \sum_{k=1}^K y_{ki}^{l*} \quad \forall i = 1, \dots, I, \forall l = 1, 2$$

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Define:

$$\begin{aligned} f^1(X^1) = & \sum_{i=1}^I \sum_{l=1}^2 p_i^l \left[\sum_{r=1}^R x_{ri}^l + \sum_{k=1}^K y_{ki}^l \right] + N \sum_{i=1}^I \left[\sum_{r=1}^R x_{ri}^2 + \sum_{k=1}^K y_{ki}^2 \right] \\ & - \gamma(x, y) - \sum_{r=1}^R \sum_{l=1}^2 (c_r^{Bl} + c_r^{Ml} + c_r^{Sl}(x_r^l)) - \sum_{r=1}^R c_r^D \\ & - \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^2 c_{ri}^l(x_{ri}^l) - \sum_{k=1}^K c^{OD} \\ & - \sum_{k=1}^K \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) - \sum_{i=1}^I P_i^l(\rho_i^l) \left[\sum_{r=1}^R d_{ri} + \sum_{k=1}^K d_{ki} \right], \end{aligned}$$

where $X^1 = (x, d^1) \in K^1 \subseteq \mathbb{R}^{I(R+1)}$

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Theorem

A vector $X^{1*} \in K^1$ is an *optimal solution* to the Delivery Company Optimization Problem if and only if is a solution to the Variational Inequality:

Find $X^{1*} \in K^1$ such that:

$$\langle \nabla f^1(X^{1*}), X^1 - X^{1*} \rangle \geq 0, \quad \forall X^1 \in K^1$$

Moreover, the variational inequality *admits at least one solution*.

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Theorem

Suppose that, for each customer i , for each regular vehicle r , for each occasional driver k and for each mean of transport l , the cost functions c_r^{Sl} , c_{ri}^l , c_{ki}^l , P_i^l constitute families of **strictly monotone functions** w.r.t. their own variables. Moreover, suppose that, the depot management cost function γ is additive, that is:

$$\gamma(x, y) = \gamma^1(x) + \gamma^2(y)$$

and that γ is a strictly convex function. Then, the operator $F^1(X^1) = \nabla f^1(X^1)$ is a strictly monotone function and the **solution** to the variational inequality is **unique**.

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ODs' VI

Define:

$$\begin{aligned} f_k^2(X_k^2) = & \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) - \sum_{l=1}^2 \tilde{c}_k^{Ml} - \sum_{l=1}^2 \tilde{c}_k^{Sl}(y_k^l) \\ & - \sum_{i=1}^I \sum_{l=1}^2 \tilde{c}_{ki}^l(y_{ki}^l) - \alpha_k \sum_{i=1}^I v_i d_{ki} \end{aligned}$$

where $X_k^2 = (y_k, d_k^2) \in K_k^2 \subseteq \mathbb{R}^{3I}$

Theorem

A vector $X^{2*} = (X_k^{2*})_{k=1,\dots,K} \in K^2 = \prod_{k=1}^K K_k^2 \subseteq \mathbb{R}^{3KI}$ is an optimal solution to the optimization problems if and only if it is a solution to the variational inequality:

Find $X^{2*} \in K^2$ such that:

$$\sum_{k=1}^K \sum_{i=1}^I \sum_{l=1}^2 \left[\frac{\partial \tilde{c}_{ki}^l(y_{ki}^{l*})}{\partial y_{ki}^l} + \frac{\partial \tilde{c}_k^{SI}(y_k^{l*})}{\partial y_{ki}^l} - \frac{\partial c_{ki}^l(y_{ki}^{l*})}{\partial y_{ki}^l} - p_i^l - \delta_l N \right] \times (y_{ki}^l - y_{ki}^{l*}) + \sum_{k=1}^K \sum_{i=1}^I [\alpha_k V_i] \times (d_{ki} - d_{ki}^*) \geq 0, \quad \forall X^2 \in K^2,$$

where $\delta_l = 1$ if $l = 2$, $\delta_l = 0$ otherwise. Moreover, such a variational inequality admits at least one solution.

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Define:

$$X^3 = (x, y, \rho) \in K^3 := \mathbb{R}^{I(RL+KL+1)}$$

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Theorem

A vector $X^{3*} \in K^3$ satisfies the *equilibrium conditions* if and only if it is a *solution to the variational inequality*:

Find $X^{3*} \in K^3$ such that:

$$\begin{aligned} & \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^2 [p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^{l*})d_{ri}^{l*} - \rho_i^{l*}] \times (x_{ri}^l - x_{ri}^{l*}) \\ & + \sum_{k=1}^K \sum_{i=1}^I \sum_{l=1}^2 [p_i^l + \hat{c}_{ki}^l(y_{ki}^{l*}) - P_i^l(\rho_i^{l*})d_{ki}^{l*} - \rho_i^{l*}] \times (y_{ki}^l - y_{ki}^{l*}) \\ & + \sum_{i=1}^I \left[\sum_{r=1}^R x_{ri}^{l*} + \sum_{k=1}^K y_{ki}^{l*} - \phi_i^l(\rho_i^{l*}) \right] \times (\rho_i^l - \rho_i^{l*}) \geq 0, \quad \forall X^3 \in K^3 \end{aligned}$$

Moreover, such a variational inequality admits *at least one solution*.

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Definition (Network Equilibrium)

An *equilibrium state of the presented network* is reached if the flows between the tiers of the network coincide and the product shipments and prices satisfy the sum of the optimality conditions.

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Theorem

A vector $X^* = (X^{1*}, X^{2*}, X^{3*}) \in \mathcal{K}$ is an *equilibrium state of the network* if and only if it is a *solution to the VI given by the sum of the previous VIs*, where the feasible set is

$$\mathcal{K} := \left\{ X \in \mathbb{R}^{I(RL+R+KL+K+1)} : \text{all the constraints are satisfied} \right\}.$$

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Theorem

A vector $X^* = (X^{1*}, X^{2*}, X^{3*}) \in \mathcal{K}$ is an *equilibrium state of the network* if and only if it is a *solution to the VI given by the sum of the previous VIs*, where the feasible set is

$$\mathcal{K} := \left\{ X \in \mathbb{R}^{I(RL+R+KL+K+1)} : \text{all the constraints are satisfied} \right\}.$$

Theorem

Under the previous assumptions, the *solution* to the total VI is *unique*.

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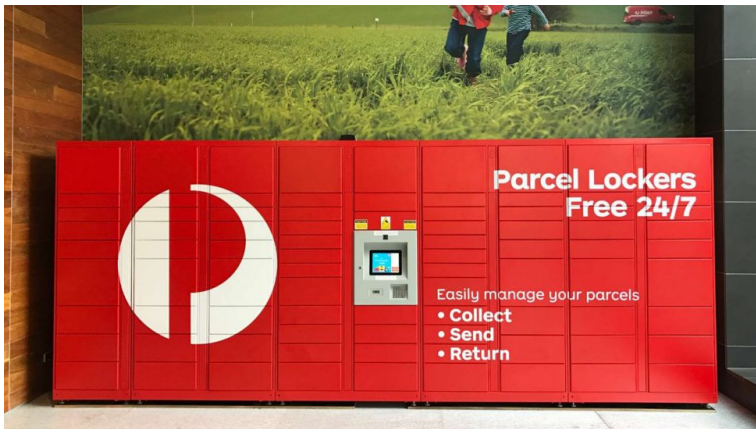
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Objective:

Lockers should be placed in **areas with high foot traffic**, such as shopping districts, metro stations, university campuses. Our aim is to **minimize the total cost** of a parcel delivery company, by establishing whether to open new lockers and to close other existing lockers in a given limited geographic area.

$$\min \sum_{i=1}^n [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i) - R_i Q_i \rho_i]$$

Costs:

installation cost of new lockers, **management cost** of all installed lockers, both existing and newly opened ones, **closure/removal cost** of closed lockers, **delivery cost** for couriers at demand nodes, which depend on the demand at each node and the percentage of customers who do not use the lockers, and the **revenue**.

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Constraints:

$$x_i, y_i, z_{ik} \in \{0, 1\}, \quad \rho_i \in [0, 1] \quad \forall i, k = 1, \dots, n.$$

$$x_i \leq (1 - l_i), \quad \forall i = 1, \dots, n;$$

$$y_i \leq l_i, \quad \forall i = 1, \dots, n;$$

$$\rho_i \geq p_{ik} \cdot (l_k + x_k - y_k), \quad \forall i, k = 1, \dots, n;$$

$$\rho_i \leq (1 - z_{ik}) + [p_{ik} \cdot (l_k + x_k - y_k)], \quad \forall i, k = 1, \dots, n;$$

$$\sum_{k=1}^n z_{ik} \geq 1, \quad \forall i = 1, \dots, n$$

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Constraints:

- If there is already an **open locker** at a given node, opening another one is not allowed.
- A locker at a given node can only be **closed** if it was previously open
- There is a lower and an upper bound for the value of the **percentage of customers** who go to the nearest open locker

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Objectives:

- Determine which **nodes/customers** are **visited by each drone** (and in which launch) and which are **visited by the truck**;

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Objectives:

- Determine which **nodes/customers** are **visited by each drone** (and in which launch) and which are **visited by the truck**;
- Identify the specific kilometer at which the **drone is launched** and retrieved;

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Objectives:

- Determine which **nodes/customers** are **visited by each drone** (and in which launch) and which are **visited by the truck**;
- Identify the specific kilometer at which the **drone is launched** and retrieved;
- Establish the **exact route of the truck**;

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Objectives:

- Determine which **nodes/customers** are **visited by each drone** (and in which launch) and which are **visited by the truck**;
- Identify the specific kilometer at which the **drone is launched** and retrieved;
- Establish the **exact route of the truck**;
- Specify the kilometer at which the **drone's battery replacement** occurs;

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- Determine which **nodes/customers** are **visited by each drone** (and in which launch) and which are **visited by the truck**;
- Identify the specific kilometer at which the **drone is launched** and retrieved;
- Establish the **exact route of the truck**;
- Specify the kilometer at which the **drone's battery replacement** occurs;

while

- Minimizing **costs and emissions**;

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- Determine which **nodes/customers** are **visited by each drone** (and in which launch) and which are **visited by the truck**;
- Identify the specific kilometer at which the **drone is launched** and retrieved;
- Establish the **exact route of the truck**;
- Specify the kilometer at which the **drone's battery replacement** occurs;

while

- Minimizing **costs and emissions**;
- Minimizing the **penalty associated with delays**;

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Objectives:

- Determine which **nodes/customers** are **visited by each drone** (and in which launch) and which are **visited by the truck**;
- Identify the specific kilometer at which the **drone is launched** and retrieved;
- Establish the **exact route of the truck**;
- Specify the kilometer at which the **drone's battery replacement** occurs;

while

- Minimizing **costs and emissions**;
- Minimizing the **penalty associated with delays**;
- Minimizing the **waiting times** (which incur costs) for the truck and drones during recovery.

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$$\begin{aligned} \min & \left\{ \sum_{i,j} \left[t(p_{ij}^T) + \sum_{n,d} t(p_{ij}^{nd,(D)}) + \sum_{n,d,k} \left[t(d_{ijk}^{(L)}(x_{ijk}^{nd})) + t(d_{ijk}^{(R)}(y_{ijk}^{nd})) \right] \right] + \sum_{n,d} \left[\gamma^{(T)} a_{nd}^{(T)} + \gamma^{(D)} a_{nd}^{(D)} \right] \right. \\ & + \sum_{i,j} \left[\alpha^{(T)} (p_{ij}^{(T)}) + \sum_{n,d} \alpha^{(D)} (p_{ij}^{nd,(D)}) + \sum_{n,d,k} \left[\alpha^{(D)} (d_{ijk}^{(L)}(x_{ijk}^{nd})) + \alpha^{(D)} (d_{ijk}^{(R)}(y_{ijk}^{nd})) \right] \right] + \sum_i \beta_i r_i \\ & + M \left[\sum_k \sqrt{\left(\sum_{i,j,n,d} (x_{ijk}^{nd})^2 \right) \left(\sum_{i,j} z_{ij}^2 \right)} + \epsilon + \sqrt{\sum_{i,j,k,n,d} [(x_{ikj}^{nd})^2 (1 - \bar{x}_{ijk}^{nd})^2]} + \epsilon \right. \\ & + \sqrt{\sum_{i,j,k,n,d} [(y_{ikj}^{nd})^2 (1 - \bar{y}_{ijk}^{nd})^2]} + \epsilon + \sum_k \sqrt{\left(\sum_{i,j,n,d} (y_{ijk}^{nd})^2 \right) \left(\sum_i z_{ik}^2 \right)} + \epsilon \\ & \left. + \sum_k \sqrt{\left(\sum_{i,n,d} (w_{ik}^{nd})^2 \right) \left(\sum_i z_{ik}^2 \right)} + \epsilon + \sqrt{\sum_{i,j} z_{ij}^2 \cdot \bar{z}_{ij}^2} + \epsilon + \sqrt{\sum_{i,j,n,d} (w_{ij}^{nd})^2 (\tilde{w}_{ij}^{nd})^2} + \epsilon \right] \left. \right\} \end{aligned}$$

Costs:

transportation costs, which result from the sum of the costs associated with both trucks and drones, **environmental impact**, **delivery delay penalties**, **waiting times**, which are expressed in cost terms through appropriate weighting coefficients.

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$$\sum_{ijk} (x_{ijk}^{nd} + y_{ijk}^{nd}) + \sum_i \left(\sum_{n,d} w_{ik}^{nd} + z_{ik} \right) \geq 0, \quad \forall i, j$$

$$(1 - \tilde{z}_{ij}) d_{ij}^{(T)} = p_{ij}^{(T)}, \quad \forall i, j$$

$$z_{ij} d_{ij}^{(T)} = p_{ij}^{(T)}, \quad \forall i, j$$

$$w_{ij}^{nd} \cdot d_{ij}^{(D)} = p_{ij}^{nd,(D)}, \quad \forall i, j, n, d$$

$$(1 - \tilde{w}_{ij}^{nd}) \cdot d_{ij}^{(D)} = p_{ij}^{nd,(D)}, \quad \forall i, j, n, d$$

$$\sum_i z_{ij} = \sum_k z_{jk}, \quad \forall j$$

$$\sum_{i,k} \tilde{x}_{ikj}^{nd} + \sum_i w_{ij}^{nd} = \sum_k w_{jk}^{nd} + \sum_{i,k} \tilde{y}_{ikj}^{nd}, \quad \forall j, n, d$$

$$\tilde{x}_{ikj}^{nd} \in [0, 1], \quad \forall i, k, j$$

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$$\tilde{x}_{ikj}^{nd} \leq x_{ikj}^{nd} \cdot G, \quad \forall i, k, j, n, d$$

$$\tilde{y}_{ikj}^{nd} \in [0, 1], \quad \forall i, k, j, n, d$$

$$\tilde{y}_{ikj}^{nd} \leq y_{ikj}^{nd} \cdot G, \quad \forall i, k, j, n, d$$

$$\sum_{i,j,k} x_{ijk}^{nd} D_k + \sum_{i,j} w_{ij}^{nd} D_j \leq C^d, \quad \forall n, d$$

$$\begin{aligned} a_{nd}^{(T)} \geq & \left[\sum_{ijk} t \left(d_{ijk}^{(L)} (x_{ijk}^{nd}) \right) + \sum_{i,j} w_{ij}^{nd} t \left(p_{ij}^{nd, (D)} \right) + \sum_{i,j,k} t \left(d_{ijk}^{(R)} (y_{ijk}^{nd}) \right) \right] \\ & - \left[\sum_{i,j,k} \left[\tilde{x}_{ijk}^{nd} t \left(p_{ij}^{(T)} - x_{ij}^{nd} \right) + \sum_{\hat{i}, \hat{j}, \hat{k}} \left[\tilde{y}_{ijk}^{nd} t \left(y_{ijk}^{nd} - p_{ij}^{(T)} \right) + \sum_{j \leq \hat{i}, \hat{j} \leq \hat{i}} t \left(p_{ij}^{(T)} \right) \right. \right. \right. \\ & \left. \left. \left. + \sum_{d \neq d} \sum_{\bar{n}} \sum_{j \leq \hat{i}, \hat{j}, \hat{k} \leq \hat{i}} \left[t_{\bar{n}d}^{(L)} \tilde{x}_{ijk}^{\bar{n}} + \left[t_{\bar{n}d}^{(R)} + a_{\bar{n}d}^{(R)} \right] \tilde{y}_{ijk}^{\bar{n}} \right] \right] \right] \right], \quad \forall n, d \end{aligned}$$

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$$\begin{aligned}
 & \sum_k z_{ki} \left[\sum_{j \leq i-1, l \leq i} \left[t \left(p_{jl}^{(T)} \right) + l_{jl}^{(T)} + \sum_{n, d, m} \left[t_{nd}^{(L)} \tilde{x}_{jlm}^{nd} + \left(t_{nd}^{(R)} + a_{nd}^{(T)} \tilde{y}_{jlm}^{nd} \right) \right] \right] \right. \\
 & + \sum_{n, d, j, l} \tilde{x}_{jli}^{nd} \left[\sum_{\bar{j} \leq j-1, \bar{l} \leq j} \left[t \left(p_{\bar{j}\bar{l}}^{(T)} \right) + l_{\bar{j}\bar{l}}^{(T)} + \sum_{\bar{n}, \bar{d}, n} \left(t_{\bar{n}\bar{l}}^{(L)} \tilde{x}_{\bar{j}l n} + \left(t_{\bar{n}\bar{d}}^{(R)} + a_{\bar{n}\bar{d}}^{(T)} \right) \tilde{y}_{\bar{j}l m}^{\bar{n}\bar{d}} + l_{\bar{n}\bar{d}} \right) \right] \right. \\
 & \left. + t \left(x_{jli}^{nd} \right) + t_{nd}^{(L)} + t \left(d_{jli}^{(L)} \left(x_{jli}^{nd} \right) \right) \right] \\
 & + \sum_{n, d, j} w_{ji}^{nd} \left[\sum_{\bar{j} \leq i-1, l \leq i} \left[t \left(p_{\bar{j}l}^{(T)} \right) + l_{\bar{j}l}^{(T)} + \sum_{\bar{n}, \bar{d}, n} \left(t_{\bar{n}l}^{(L)} \tilde{x}_{\bar{j}l n} + \left(t_{\bar{n}\bar{d}}^{(R)} + a_{\bar{n}\bar{d}}^{(T)} \right) \tilde{y}_{\bar{j}l m}^{\bar{n}\bar{d}} + l_{\bar{n}\bar{d}} \right) \right] \right. \\
 & \left. + \tilde{x}_{\bar{j}li}^{nd} \left[t \left(x_{\bar{j}li}^{nd} \right) + t_{nd}^{(L)} + t \left(d_{\bar{j}li}^{(L)} \left(x_{\bar{j}li}^{nd} \right) \right) \right] + t \left(p_{\bar{j}l}^{nd, (D)} + l_{\bar{j}l}^{nd} \right) \right] \left. \right] - r_i \leq \bar{T}_i, \quad i
 \end{aligned}$$

$$a_{nd}^{(D)} \geq -a_{nd}^{(T)}, \quad \forall n, d$$

$$\sum_{i, j, k} \tilde{x}_{ijk}^{nd} \leq 1, \quad \forall n, d$$

$$\sum_{i, j, k} \tilde{x}_{ijk}^{nd} = \sum_{\bar{i}, \bar{j}, \bar{k}} \tilde{y}_{\bar{i}\bar{j}\bar{k}}^{nd}, \quad \forall n, d$$

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Constraints:

- Every customer node must be **visited exactly once**.
- If the truck arrives at a node, it must also **depart** from that **same** node.
- For each drone launch, the drone's carried load must not exceed its **maximum capacity**.
- Every launched **drone** must be retrieved.
- A **drone** can be **launched** on an arc only if the **truck** actually **travels** along that arc.
- A **drone** can be **recovered** along an arc only if the **truck travels** along that arc.
- Each drone can perform **multiple deliveries** with the same battery until its energy is depleted.
- ...

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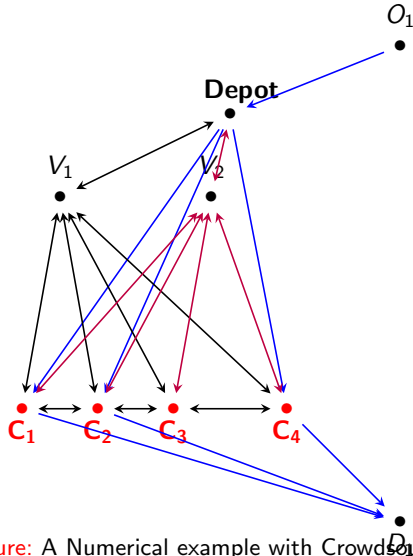


Figure: A Numerical example with Crowdsourcing

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Data

- 2 RDs with a petrol/diesel vehicle and an electric vehicle
- $\bar{C}_1^1 = 100$, $\bar{C}_1^2 = 70$, $\bar{C}_2^1 = 150$, $\bar{C}_2^2 = 90$
- 1 occasional driver with an electric vehicle
- $\bar{C}_k^2 = 60$
- 4 customers
- $\beta_{13}^1 = \beta_{13}^2 = 0$, $\beta_{11}^I = \beta_{12}^I = \beta_{14}^I = 1$

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Two Simulations

- S1:** without occasional drivers;
- S2:** with occasional driver.

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S1	$i = 1$	$i = 2$	$i = 3$	$i = 4$	Objective Function
x_{1i}^{1*}	0	0	0	0	
x_{1i}^{2*}	18.84	8.69	24.52	17.95	
x_{2i}^{1*}	0	0	0	0	
x_{2i}^{2*}	21.16	11.31	15.48	12.05	
y_{1i}^{1*}	-	-	-	-	11,728
y_{1i}^{2*}	-	-	-	-	

Table: Optimal solutions in S1

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x_{2i}^{2*}	21.16	11.31	15.48	12.05	
y_{1i}^{1*}	-	-	-	-	11,728
y_{1i}^{2*}	-	-	-	-	

Table: Optimal solutions in S1

S2	$i = 1$	$i = 2$	$i = 3$	$i = 4$	Objective Function
x_{1i}^{1*}	0	0	0	0	
x_{1i}^{2*}	5.53	2.66	26.44	11.70	
x_{2i}^{1*}	0	0	0	0	
x_{2i}^{2*}	5.33	2.55	13.56	5.87	
y_{1i}^{1*}	-	-	-	-	12,150
y_{1i}^{2*}	29.14	14.79	0	12.43	

Table: Optimal solutions in S2

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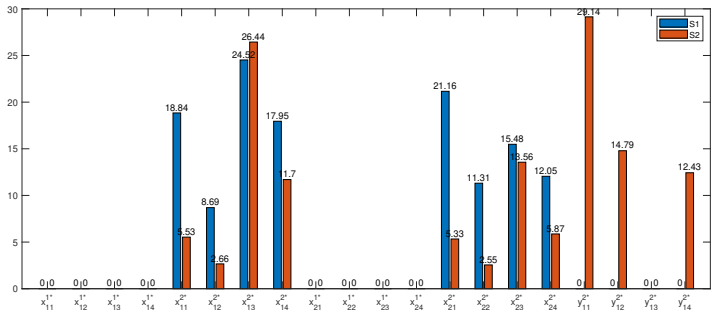


Figure: Comparison of optimal solutions in S1 and S2

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THANK YOU
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