



How Parcel Lockers and Crowd-Sourcing Shape Urban Delivery Systems

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- University of Naples Federico II
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- University of Catania

Outline

University of
Catania

Patrizia
DANIELE

Outline

1 Urban Delivery System

Outline

University of
Catania

Patrizia
DANIELE

Outline

1 Urban Delivery System

2 Parcel Lockers

- The Mathematical Model
- Model with different sizes

Outline

University of
Catania

Patrizia
DANIELE

Outline

1 Urban Delivery System

2 Parcel Lockers

- The Mathematical Model
- Model with different sizes

3 Crowd-Sourcing Delivery System

- Benefits and Examples
- The Game-Theoretical Model
- Numerical Simulations

E-commerce

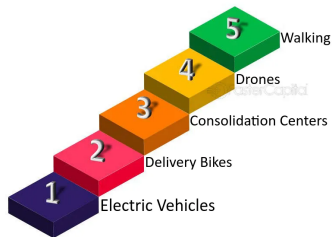
- The rapid growth of **e-commerce** has intensified the need for efficient last-mile delivery solutions



Environmental concerns

- Rise of **environmental concerns** and the emergence of **new business models** have prompted companies to explore alternative strategies to optimize last-mile delivery

Environmental Impact of Last-Mile Delivery



Green Vehicles

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Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

- One promising approach is the integration of **green vehicles** into the last-mile delivery process (represent a good **alternative** to petrol and diesel-powered fleets, reduce **carbon emissions** and reduce **operational costs**)



Crowdsourcing

- Another approach is the integration of **crowdsourced drivers** into the last-mile delivery process



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Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Regular Vehicles and Occasional Drivers

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Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

- We develop a **model** to optimize last-mile delivery operations using a **combination of RVs and ODs** (individuals who can transport goods as part of their **daily routines**). Also this approach results in significant **cost reductions and increased efficiency**



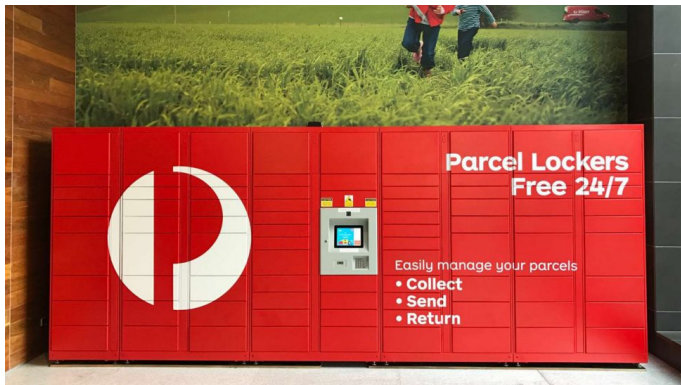
Truck-drone Systems

- Another solution is the **truck-drone system** (which typically consists of **delivery trucks** which serve as **mobile hubs** from which drones are deployed to deliver packages to customers).



Parcel Lockers

- Another approach is the use of **parcel lockers** which eliminate the inconvenience of missed deliveries and restrictive delivery schedules.



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University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Advantages:

They are very useful in **urban logistics** and provide numerous advantages for consumers and companies.

For **consumers**:

- eliminate the inconvenience of **missed deliveries** and restrictive delivery schedules
- provide **24/7 accessibility**
- reduce **dependency on home delivery**

Parcel Lockers

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

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For **consumers**:

- eliminate the inconvenience of **missed deliveries** and restrictive delivery schedules
- provide **24/7 accessibility**
- reduce **dependency on home delivery**

For **companies**, like Amazon:

- contribute to **cost reduction**, since multiple deliveries into a single location are allowed
- minimize **failed delivery attempts**
- reduce **carbon emissions**

Remarks:

- The effectiveness of parcel lockers depends on their **strategic placement**.
- Many factors must be considered to optimize their deployment: analyzing **consumer behavior** and **demographics** helps determine the most efficient locations, ensuring that lockers are positioned where demand is highest.
- **Traffic flow** and **accessibility** play a crucial role, as lockers should be placed in areas with high foot traffic, such as shopping districts, metro stations, and university campuses.

Outline

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

1 Urban Delivery System

2 Parcel Lockers

- The Mathematical Model
- Model with different sizes

3 Crowd-Sourcing Delivery System

- Benefits and Examples
- The Game-Theoretical Model
- Numerical Simulations

The Mathematical Model

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Objective:

The optimization model aims to **minimize the total operational cost (including installation, management, and removal costs, as well as delivery costs and revenue flows)** incurred by a parcel delivery company through strategic decisions on the installation of new parcel lockers and the decommissioning of existing ones within a defined urban area.

The Mathematical Model

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

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Assume we have n nodes, which may correspond either to a location where a parcel locker already exists or to a potential site where a locker could be installed.

The Mathematical Model

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Decision Variables:

$$x_i = \begin{cases} 1 & \text{if a locker needs to be opened/installed at node } i \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if a locker at node } i \text{ needs to be closed/removed} \\ 0 & \text{otherwise} \end{cases}$$

Parameters:

- $\rho_i \in [0, 1]$: **percentage of customers** at node i who go to the nearest locker (this variable depends on the distance to the nearest open locker)

The Optimization Problem

University of
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Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

$$\min \sum_{i=1}^n [F_i x_i]$$

installation costs of new lockers

The Optimization Problem

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

$$\min \sum_{i=1}^n [F_i x_i + G_i \cdot (x_i + l_i - y_i)]$$

management cost of all installed lockers, both existing and new ones

The Optimization Problem

University of
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Patrizia
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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

$$\min \sum_{i=1}^n [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i]$$

closure/removal cost of closed lockers

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University of
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Patrizia
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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

$$\min \sum_{i=1}^n [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i)]$$

delivery cost for couriers at demand nodes

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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

$$\min \sum_{i=1}^n [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i) - R_i Q_i \rho_i]$$

revenue (to be maximized) from orders picked up by customers at the lockers

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University of
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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

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Constraints:

$$x_i \leq (1 - l_i), \quad \forall i = 1, \dots, n;$$

If there is already an open locker at a given node, opening another one is not allowed

The Optimization Problem

University of
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Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

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Constraints:

$$x_i \leq (1 - l_i), \quad \forall i = 1, \dots, n;$$

$$y_i \leq l_i, \quad \forall i = 1, \dots, n;$$

A locker at a given node can only be closed if it was previously open (i.e., a locker already existed at that node)

The Optimization Problem

University of
Catania

Patrizia
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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

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Constraints:

$$x_i \leq (1 - l_i), \quad \forall i = 1, \dots, n;$$

$$y_i \leq l_i, \quad \forall i = 1, \dots, n;$$

$$\rho_i \geq p_{ik} \cdot (l_k + x_k - y_k), \quad \forall i, k = 1, \dots, n;$$

$$\rho_i \leq (1 - z_{ik}) + [p_{ik} \cdot (l_k + x_k - y_k)], \quad \forall i, k = 1, \dots, n;$$

lower and upper bounds for the value of the maximum percentage of customers who go to the nearest open locker

The Optimization Problem

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

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$$\rho_i \leq (1 - z_{ik}) + [p_{ik} \cdot (l_k + x_k - y_k)], \quad \forall i, k = 1, \dots, n;$$

$$\sum_{k=1}^n z_{ik} \geq 1, \quad \forall i = 1, \dots, n;$$

$$x_i, y_i, z_{ik} \in \{0, 1\}, \quad \rho_i \in [0, 1] \quad \forall i, k = 1, \dots, n.$$

Domain of the variables

Numerical Simulations

University of
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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

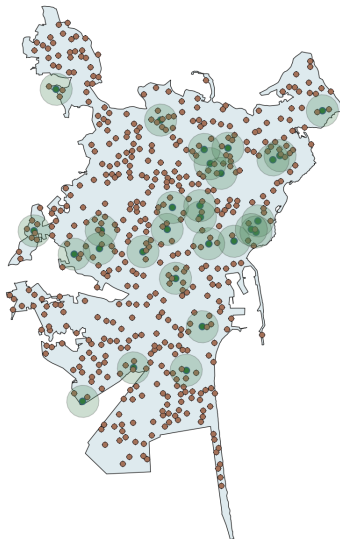
Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations



Numerical Simulations

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Costs:

- **fixed installation cost** for opening a new locker was set to 1,500 € per site ($F_i = 1500$)
- **cost of decommissioning** an existing locker was assumed to be 1,000 € ($D_i = 1000$)
- **unit delivery cost** per order was set at 2.50 € ($C_i = 2.5$)
- **revenue** generated per successful delivery was set at 7.00 € ($R_i = 7$)

Numerical Simulations

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Optimal Solution

- **computational time:** 23.43 seconds
- **optimized total cost:** 199,243.95 €
- 92 **new lockers** at selected demand nodes
- 6 existing **lockers decommissioned**
- final network configuration: **111 active lockers**

Numerical Simulations

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

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Comparison

the existing locker configuration provided service to only **110 out of 500** demand nodes, representing 22% coverage within a 400-meter service radius.

After optimization, the locker network achieved **full coverage**, effectively serving all 500 demand nodes and thereby attaining a 100% coverage rate.

Optimal Spatial Distribution

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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

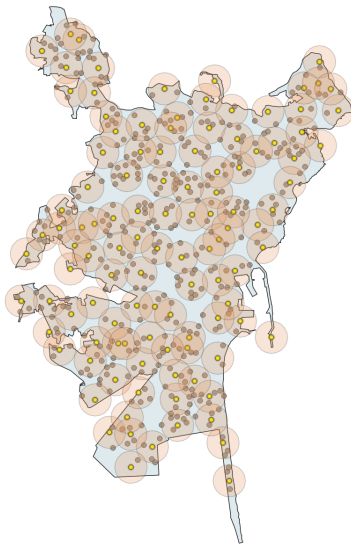
Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations



Outline

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

1 Urban Delivery System

2 Parcel Lockers

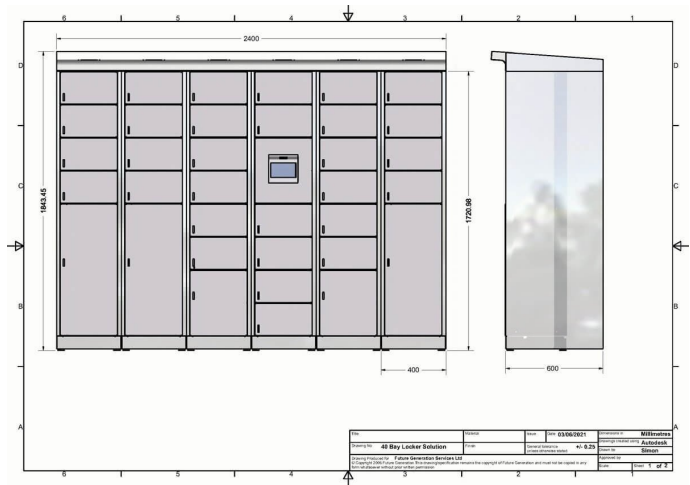
- The Mathematical Model
- Model with different sizes

3 Crowd-Sourcing Delivery System

- Benefits and Examples
- The Game-Theoretical Model
- Numerical Simulations

Model with different sizes

- Using parcel lockers of **different** sizes is more convenient because it **avoids wasting space** and makes it easier to **handle** both small and large packages efficiently.



The Mathematical Model

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Decision Variables:

- $x_{ij} \in \mathbb{R}_+$: fraction of a Standard Module of type j installed at node i ;
- $y_{ij} \in [0, 1]$: fraction of the existing locker of type j at node i to be decommissioned;

Parameters:

- K_j : maximum delivery capacity of a locker of type j ;
- N_j : total number of slots contained in one full standard unit (rack) of type j ;
- W_i : maximum capacity of all the modules at node i .

The Optimization Model

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Objective Function

Minimize the total system costs associated with the parcel locker network: **installation costs** for new modular units, **management expenses**, **decommissioning costs** related to existing locker sites.

The Optimization Model

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Objective Function

Minimize the total system costs associated with the parcel locker network: **installation costs** for new modular units, **management expenses**, **decommissioning costs** related to existing locker sites.

Constraints

- the total physical dimension of all installed modules at node i does not exceed a predefined and allowed maximum capacity;
- the total volume of parcels assigned to a locker at node i does not exceed its physical storage capacity.
- . . .

Outline

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

1 Urban Delivery System

2 Parcel Lockers

- The Mathematical Model
- Model with different sizes

3 Crowd-Sourcing Delivery System

- **Benefits and Examples**
- The Game-Theoretical Model
- Numerical Simulations

Crowd-Sourcing Delivery System

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Benefits:

- Reduced operational costs
- Increased delivery flexibility
- Environmental benefits
- Job creation opportunities

Real-World Examples:

- Amazon Flex: \$18–\$25/hour
- Walmart: Fast delivery program
- Food delivery: Deliveroo, JustEat, Glovo

Outline

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

1 Urban Delivery System

2 Parcel Lockers

- The Mathematical Model
- Model with different sizes

3 Crowd-Sourcing Delivery System

- Benefits and Examples
- The Game-Theoretical Model
- Numerical Simulations

The Model Framework: Players and Strategies

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Tripartite evolutionary game model.

Players

- Logistics Providers (LP)
- Independent Drivers (ID)
- Consumers (C)

Strategies

- LP: Regular crowd-sourcing (RC) vs Occasional (OC)
- ID: Cooperate (CP) vs Not cooperate (NCP)
- C: Express delivery (ED) vs Standard delivery (SD)

Key Features of the Model

University of
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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

- Evolutionary game framework (no full rationality)
- Introduction of a soft reputation mechanism:
 - Bonus for trustworthy drivers
 - Penalty when untrustworthy drivers dominate
- Incentives shared among players:
 - Revenue-sharing
 - Cost-sharing
- Compensation for delays

Goal: study long-run behavioral outcomes

Model Parameter

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Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Parameter	Description	Value
π_L^+	Revenue of L under crowd-sourcing	$\pi_L^+ > 0$
π_L	Revenue of L under occasional crowd-sourcing	$\pi_L > 0$
π_D	Revenue of D under independent delivery	$\pi_D > 0$
R	Crowd-sourcing delivery service revenue	$R > 0$
π_C	Satisfaction benefit of C	$\pi_C > 0$
π_C^+	Additional satisfaction benefit of C under regular crowd-sourcing	$\pi_C^+ > 0$
c_t	Transportation cost for standard home-delivery	$c_t > 0$
c_e	Transportation cost for express home-delivery	$c_e > c_t$
c_p	Cost of L for using crowd-sourcing platform	$c_p > 0$
c^+	Additional cost of C for express delivery	$c^+ > 0$
S	Environmental benefit of C for crowd-sourcing	$S > 0$
d	Late delivery compensation paid by L to C	$d > 0$
T	Proportion of trustworthy drivers registered on the platform	$0 < T < 1$
p	Penalty term for D under $1 - T$ untrustworthy drivers	$p > 0$
B	Bonus for D under T trustworthy drivers	$B > 0$
l_L	Potential loss of L for inefficiency	$l_L > 0$
l_C	Potential loss of C for inefficiency	$l_C > 0$
α	Revenue-sharing parameter	$0 < \alpha < 1$
β	Cost-sharing parameter	$0 < \beta < 1$

Payoff Matrix

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Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

		C	
L	D	ED z	SD $1 - z$
RC x	CP y	$\pi_L^+ + \alpha R - \beta c_e - c_p - d$ $(1 - \alpha)R - (1 - \beta)c_e + BT - p(1 - T)$ $\pi_C + \pi_C^+ + S + d - c^+$	$\pi_L^+ + \alpha R - \beta c_t - c_p$ $(1 - \alpha)R - (1 - \beta)c_t + BT - p(1 - T)$ $\pi_C + \pi_C^+$
	NCP $1 - y$	$\pi_L^+ + R - c_e - c_p - d$ π_D $\pi_C + d - c^+$	$\pi_L^+ + R - c_t - c_p$ π_D π_C
OC $1 - x$	CP y	$\pi_L - l_L - d$ $R - c_e + BT - p(1 - T)$ $\pi_C + d - c^+$	$\pi_L - l_L$ $R - c_t + BT - p(1 - T)$ π_C
	NCP $1 - y$	$\pi_L + R - l_L - c_e - d$ π_D $\pi_C + d - c^+ - l_C$	$\pi_L + R - l_L - c_t$ π_D π_C

Replicator Dynamics

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Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

$$\begin{aligned}\dot{x} &= x(1-x)(U_{RC} - U_{OC}) \\ &= x(1-x)(\pi_L^+ + \alpha R y + I_L - \pi_L - c_p - (1-z)\beta c_t y - \beta c_e y z), \\ \dot{y} &= y(1-y)(U_{CP} - U_{NCP}) \\ &= y(1-y)(\pi_D + (1-\alpha x)R - c_t(1-\beta x)(1-z) - c^e z(1-\beta x) \\ &\quad + BT - (1-T)p), \\ \dot{z} &= z(1-z)(U_{ED} - U_{SD}) \\ &= z(1-z)(d + Sxy - c^+ + I_C(x + y - xy - 1)).\end{aligned}$$

- Successful strategies spread
- Poorly performing strategies disappear
- Dynamic path toward stable equilibria

Equilibrium Analysis

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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

- 8 pure-strategy equilibria
- Stability via Jacobian eigenvalues
- Only two asymptotically stable equilibria

Assumptions:

$$\begin{aligned} R - c_t + BT - p(1 - T) &> 0, & R - c_e + BT - p(1 - T) &> 0, \\ \pi_L^+ + \alpha R - \beta c_t - c_p - \pi_L + l_L &> 0, & \pi_L^+ + \alpha R - \beta c_e - c_p - \pi_L + l_L &> 0, \\ \pi_L^+ - \pi_L &> 0, & d - c^+ &< 0. \end{aligned}$$

Stable Outcomes

- (1, 1, 0): RC, CP, SD
- (1, 1, 1): Full cooperation, express delivery

Outline

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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

1 Urban Delivery System

2 Parcel Lockers

- The Mathematical Model
- Model with different sizes

3 Crowd-Sourcing Delivery System

- Benefits and Examples
- The Game-Theoretical Model
- Numerical Simulations

Numerical Simulations

University of
Catania

Patrizia
DANIELE

Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

Parameters:

π_L^+	π_L	π_D	R	c_t
[2.5; 5]	[2; 5]	[2; 2.5]	[3.5; 4.3]	[1; 1.8]
c_e	c_p	c^+	p	B
[1; 1.8]	[2; 2.5]	[0.1; 2.6]	[1.8; 4.5]	[1; 3]
d	T	l_L	l_C	S
[0.2, 0.7]	[0.1, 0.9]	[0.3; 1]	[0.3; 1]	[0.1; 1]
α	β			
[0; 1]	[0; 1]			

Initial strategies: $x = 0.5$, $y = 0.5$, and $z = 0.5$.

Simulation Results Visualization

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The Mathematical
Model

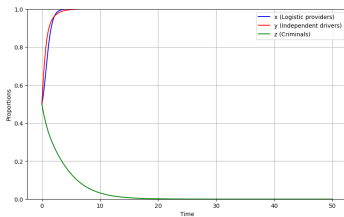
Model with different
sizes

Crowd-Sourcing
Delivery System

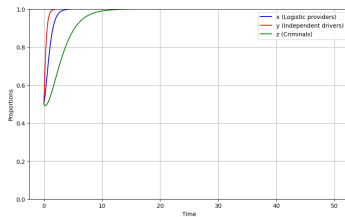
Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations



(a) Evolution to $(1, 1, 0)$ (frequency 52.23%)

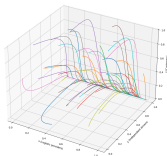


(b) Evolution to $(1, 1, 1)$ (frequency 46.63%)

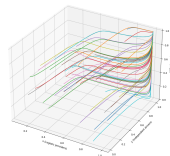
Sensitivity Analysis

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The system is robust to changes in initial probabilities.

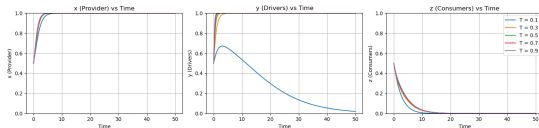


(a) Evolution to $(1, 1, 0)$



(b) Evolution to $(1, 1, 1)$

Changes in trustworthy drivers evolve to the unstable point $(1, 0, 0)$.



Strategic Insights

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Urban Delivery
System

Parcel Lockers

The Mathematical
Model

Model with different
sizes

Crowd-Sourcing
Delivery System

Benefits and
Examples

The
Game-Theoretical
Model

Numerical
Simulations

- Crowd-sourced delivery works only with aligned incentives
- Reputation stabilizes cooperation
- Evolutionary games capture adaptive behavior

Stakeholders:

- Providers: Reputation influences crowdsourcing
- Drivers: Build reputation through cooperation
- Customers: Compensation drives express choice

Policy Levers:

- $\uparrow \alpha$ or $\downarrow \beta$ sustains RC
- Reputation mechanisms enforce cooperation
- Environmental incentives
- Platform governance

Infrastructure + behavior + reputation = sustainable delivery