Mathematical Modeling of Parcel Locker Networks in Urban Delivery Systems

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Joint work with G. Colajanni - D. Sciacca

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PIANO NAZIONALE DI RIPRESA E RESILIENZA

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- University of Naples Federico II
- Institute of System Analysis and Computer Science "Antonio Ruberti" of the National Research Council (IASI-CNR)
- University of Catania

Outline

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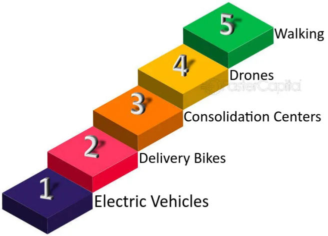
E-commerce

- The rapid growth of e-commerce has intensified the need for efficient last-mile delivery solutions

Environmental concerns

- Rise of environmental concerns and the emergence of new business models have prompted companies to explore alternative strategies to optimize last-mile delivery

Environmental Impact of Last-Mile Delivery



Green Vehicles

- One promising approach is the integration of **green vehicles** into the last-mile delivery process (represent a good alternative to petrol and diesel-powered fleets, reduce carbon emissions and reduce operational costs)

Crowdsoursing

- Another approach is the integration of crowdsourced drivers into the last-mile delivery process

Regular Vehicles and Occasional Drivers

- We develop a model to optimize last-mile delivery operations using a combination of RVs and ODs (individuals who can transport goods as part of their daily routines). Also this approach results in significant cost reductions and increased efficiency

Truck-drone Systems

- Another solution is the truck-drone system (which typically consists of delivery trucks which serve as mobile hubs from which drones are deployed to deliver packages to customers).

Truck-drone Systems

Advantages:

- Drones can handle short-distance deliveries (particularly in congested urban areas or remote rural locations)
- Significantly reduce total delivery times
- Significantly reduce energy consumption compared to conventional truck-only routes.

Parcel Lockers

- Another approach is the use of parcel lockers which eliminate the inconvenience of missed deliveries and restrictive delivery schedules.

Parcel Lockers

Advantages:

They are very useful in urban logistics and provide numerous advantages for consumers and companies.

For consumers:

- eliminate the inconvenience of missed deliveries and restrictive delivery schedules
- provide 24/7 accessibility
- reduce dependency on home delivery

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For consumers:

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For companies, like Amazon:

- contribute to cost reduction, since multiple deliveries into a single location are allowed
- minimize failed delivery attempts
- reduce carbon emissions

Literature Overview

1. Y. Liu, Q. Ye, J. Escribano-Macias, Y. Feng, E. Candela, P. Angeloudis, *Route Planning for Last-Mile Deliveries Using Mobile Parcel Lockers: A Hybrid Q-Learning Network Approach*, Transportation Research Part E, 177, 103234, 2023:
   the authors discuss route planning for last-mile deliveries with mobile parcel lockers

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   the authors discuss route planning for last-mile deliveries with mobile parcel lockers
2. M. Kahr, *Determining locations and layouts for parcel lockers to support last-mile delivery*, Omega, 113, 102721, 2022:
   the author examines how parcel lockers can support the sustainability of the last-mile supply chain

Literature Overview

1. Y. Deutsch, B. Golany, *A parcel locker network as a solution to the logistics last mile problem*, International Journal of Production Research, `https://doi.org/10.1080/00207543.2017.1395490`, 2017: the authors formulate the optimization problem of parcel locker as a 0-1 integer linear program, equivalent to the Uncapacitated Facility Location Problem, aiming at maximizing total profit while considering customer convenience, operational costs, and location efficiency.

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2. S. Mancini, M. Gansterer, C. Triki, *Locker box location planning under uncertainty in demand and capacity availability*, Omega, 120, 102910. https://doi.org/10.1016/j.omega.2023.102910,2023: the authors explore the strategic planning of parcel locker locations in last-mile logistics under uncertainty and model the problem as an extension of the capacitated facility location problem, incorporating stochastic elements to optimize placement.

Remarks:

- The effectiveness of parcel lockers depends on their strategic placement.
- Many factors must be considered to optimize their deployment: analyzing consumer behavior and demographics helps determine the most efficient locations, ensuring that lockers are positioned where demand is highest.
- Traffic flow and accessibility play a crucial role, as lockers should be placed in areas with high foot traffic, such as shopping districts, metro stations, and university campuses.

The Mathematical Model

Objective:

The optimization model aims to minimize the total operational cost (including installation, management, and removal costs, as well as delivery costs and revenue flows) incurred by a parcel delivery company through strategic decisions on the installation of new parcel lockers and the decommissioning of existing ones within a defined urban area.

The Mathematical Model

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The optimization model aims to minimize the total operational cost (including installation, management, and removal costs, as well as delivery costs and revenue flows) incurred by a parcel delivery company through strategic decisions on the installation of new parcel lockers and the decommissioning of existing ones within a defined urban area.

Assume we have n nodes, which may correspond either to a location where a parcel locker already exists or to a potential site where a locker could be installed.

The Mathematical Model

Decision Variables:

$$x_i = \begin{cases} 1 & \text{if a locker needs to be opened/installed at node } i \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if a locker at node } i \text{ needs to be closed/removed} \\ 0 & \text{otherwise} \end{cases}$$

Parameters:

- $\rho_i \in [0, 1]$: percentage of customers at node i who go to the nearest locker (this variable depends on the distance to the nearest open locker)

The Optimization Problem

$$\min \sum_{i=1}^{n} [F_i x_i$$

installation costs of new lockers

The Optimization Problem

$$\min \sum_{i=1}^{n} [F_i x_i + G_i \cdot (x_i + l_i - y_i)$$

management cost of all installed lockers, both existing and new ones

The Optimization Problem

$$\min \sum_{i=1}^{n} [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i$$

closure/removal cost of closed lockers

The Optimization Problem

$$\min \sum_{i=1}^{n} [F_i x_i + G_i \cdot (x_i + I_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i)$$

delivery cost for couriers at demand nodes

The Optimization Problem

$$\min \sum_{i=1}^{n} [F_i x_i + G_i \cdot (x_i + I_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i) - R_i Q_i \rho_i]$$

revenue (to be maximized) from orders picked up by customers at the lockers

The Optimization Problem

$$\min \sum_{i=1}^{n} [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i) - R_i Q_i \rho_i]$$

Constraints:

$$x_i \leq (1 - l_i), \quad \forall i = 1, \ldots, n;$$

If there is already an open locker at a given node, opening another one is not allowed

The Optimization Problem

$$\min \sum_{i=1}^{n} [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i) - R_i Q_i \rho_i]$$

Constraints:

$$x_i \leq (1 - l_i), \quad \forall i = 1, \dots, n;$$

$$y_i \leq l_i, \quad \forall i = 1, \dots, n;$$

A locker at a given node can only be closed if it was previously open (i.e., a locker already existed at that node)

The Optimization Problem

$$\min \sum_{i=1}^{n} [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i) - R_i Q_i \rho_i]$$

Constraints:

$$x_i \leq (1 - l_i), \quad \forall i = 1, \ldots, n;$$

$$y_i \leq l_i, \quad \forall i = 1, \ldots, n;$$

$$\rho_i \geq p_{ik} \cdot (l_k + x_k - y_k), \quad \forall i, k = 1, \ldots, n;$$

$$\rho_i \leq (1 - z_{ik}) + [p_{ik} \cdot (l_k + x_k - y_k)], \quad \forall i, k = 1, \ldots, n;$$

lower and upper bounds for the value of the maximum percentage of customers who go to the nearest open locker

The Optimization Problem

$$\min \sum_{i=1}^{n} [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i) - R_i Q_i \rho_i]$$

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$$\sum_{k=1}^{n} z_{ik} \geq 1, \quad \forall i = 1, \dots, n;$$

$$x_i, y_i, z_{ik} \in \{0, 1\}, \quad \rho_i \in [0, 1] \quad \forall i, k = 1, \dots, n.$$

Domain of the variables

Numerical Simulations

Algorithm Setup

- d_{ik}: inter-node distances, derived from official data provided by ISTAT (Italian National Institute of Statistics).
- estimation of demand at each node, based on socioeconomic indicators such as economic activity and education levels, derived from official data provided by ISTAT
- l_i: presence of pre-existing lockers, based on an empirical survey of current locker infrastructure within the metropolitan area
- $p_{ik}(d_{ik}) = \begin{cases} 1 - \dfrac{d_{ik}}{\overline{D}_i} & \text{if } 0 \leq d_{ik} \leq \overline{D}_i \\ 0 & \text{if } d_{ik} > \overline{D}_i, \end{cases}$: customer's probability of utilizing a locker, with a maximum distance threshold $\overline{D}_i = \overline{D} = 400\text{m}$

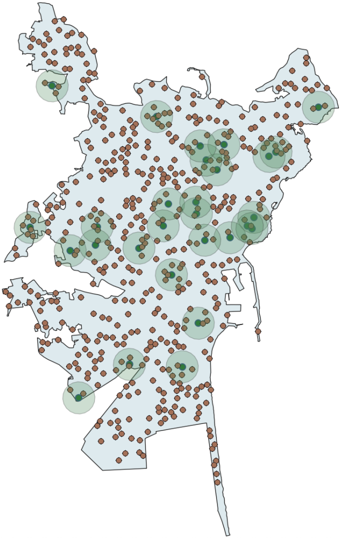
Numerical Simulations

Algorithm Setup

- $n = 500$ nodes: potential demand and locker sites within the municipal boundaries of Catania
- solver time limit, imposed and calibrated during the computational experiments

The analysis focuses on the resulting configuration of active and inactive lockers, the assignment of demand nodes to open lockers, and the achieved coverage levels (ρ_i)

Numerical Simulations



Numerical Simulations

Costs:

- fixed installation cost for opening a new locker was set to 1,500 € per site ($F_i = 1500$)
- cost of decommissioning an existing locker was assumed to be 1,000 € ($D_i = 1000$)
- unit delivery cost per order was set at 2.50 € ($C_i = 2.5$)
- revenue generated per successful delivery was set at 7.00 € ($R_i = 7$)

Numerical Simulations

Optimal Solution

- computational time: 23.43 seconds
- optimized total cost: 199,243.95 €
- 92 new lockers at selected demand nodes
- 6 existing lockers decommissioned
- final network configuration: 111 active lockers

Numerical Simulations

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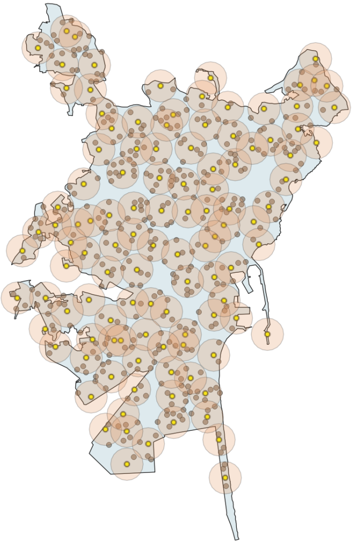
Comparison

the existing locker configuration provided service to only 110 out of 500 demand nodes, representing 22% coverage within a 400-meter service radius.
After optimization, the locker network achieved full coverage, effectively serving all 500 demand nodes and thereby attaining a 100% coverage rate.

Optimal Spatial Distribution

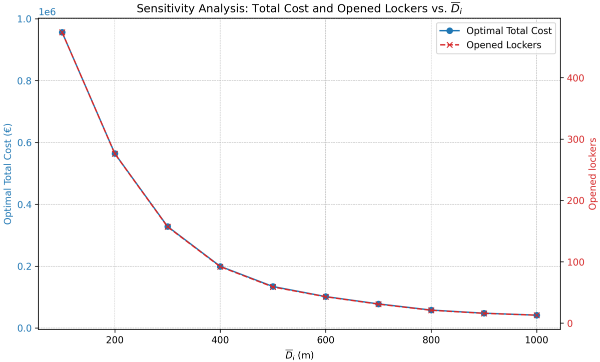


Sensitivity Analysis on $\overline{D}_i$

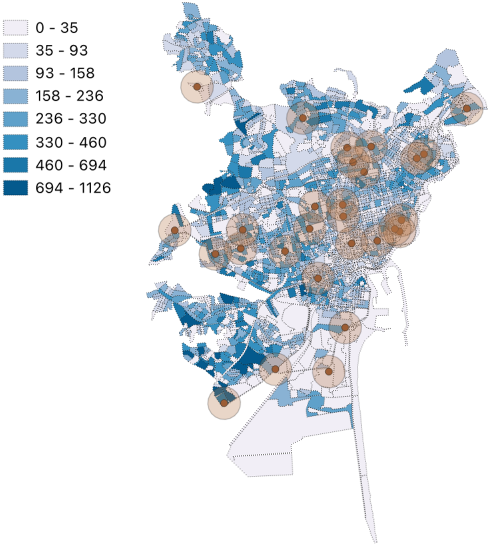
$\overline{D}_i$ (m)	Solution Status	Optimized Cost () €	Time (s)	New Lockers	Closed Lockers	Total Lockers	Fixed Cov. (%)
100	Optimal	731,862.50	0.48	474	0	499	100.0
200	Optimal	437,214.57	0.78	276	3	298	100.0
300	Optimal	258,593.54	1.07	157	8	174	100.0
400	Optimal	163,068.13	2.69	92	6	111	100.0
500	Optimal	113,474.75	4.03	59	8	76	74.0
600	Optimal	87,184.35	8.55	43	13	55	55.4
700	Optimal	70,432.16	7.88	31	11	45	45.2
800	Optimal	56,025.02	12.40	21	9	37	39.2
900	Optimal	47,813.20	14.67	17	14	28	30.0
1000	Optimal	42,054.72	23.76	13	13	25	28.0

Table: Sensitivity analysis summary for $\overline{D}_i$: total cost, locker configuration, and service coverage (including optimal coverage at $D_{\text{opt}} = 400\,\text{m}$).

Sensitivity Analysis on $\overline{D}_i$



Numerical Simulation with a tailored GA



Numerical Simulation with a tailored GA

Data:

- Centroids: $n = 2793$
- ISTAT info:
  - resident population
  - average household size
  - average net income
  - education level
- 28 existing lockers

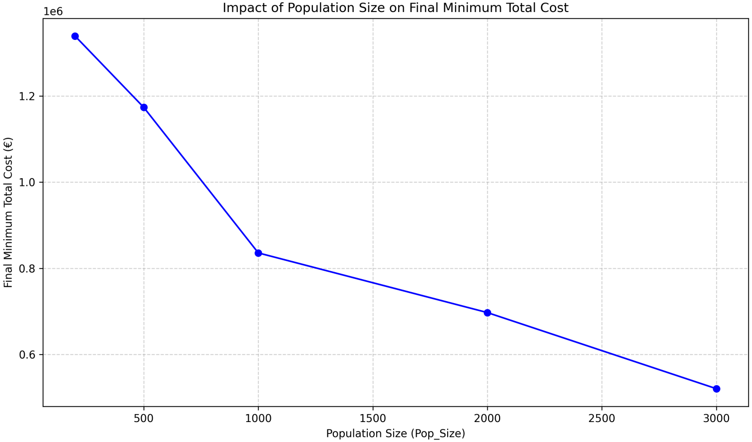
Numerical Simulation with a tailored GA

Variable Name	**Value**
`Population_Size`	1000
`Number_of_Generations`	250
`Elite_Size`	15
`Mutation_Rate`	0.18
`Crossover_Rate`	0.8

Table: GA Parameters

Population sizes	200	500	1000	2000	3000

Numerical Simulation with a tailored GA

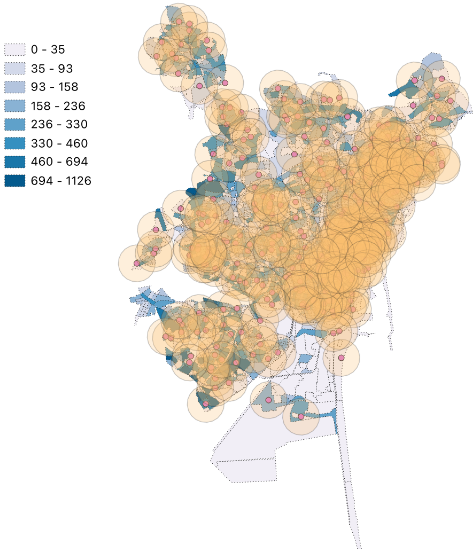


Numerical Simulation with a tailored GA

Results:

- 180 generations
- total cost reduction: from 1,953,546.41 € to 835,637.96 €
- convergence after generation 150
- 310 active lockers
- 16 closed pre-existing lockers

Numerical Simulation with a tailored GA



Comparison with a classical GA

Metric	**AGS (Standard GA)**	**AGP (Tailored GA)**
Total Cost	1,170,995.35 €	**835,637.96 €**
Node Coverage (%)	**98.32**	98.07
Active Lockers	408	**310**
Execution Time (s)	3891	**3281**

Table: Performance comparison: AGS vs. AGP

Conclusions and Further Research

- Scalable, data-driven optimization framework for parcel locker network design
- Extension of the model to dynamic settings, incorporating temporal demand uncertainty and learning mechanisms for user behavior adaptation
- Integration of environmental criteria, such as carbon emissions or energy consumption
- Hybrid optimization techniques that combine metaheuristics with exact or decomposition methods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