A Variational Formulation for a Human Migration Problem

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Joint paper with G. Cappello

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Outline

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Definition (Human Migration)

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Main Causes

Many social and economical factors affect the dynamics of human populations, such as poverty, violence, war, dictatorships, persecutions, oppression, genocide, ethnic cleansing, climate change, tsunamis, floods, earthquakes, famines, family reunification as well as economic and educational possibilities or a job.

Introduction

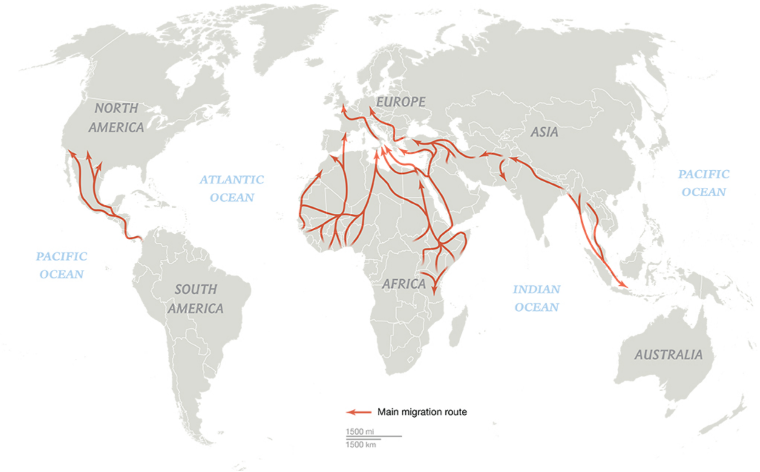

Figure: World's congested human migration routes

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From 2018 until January 2019, 17% of arrivals by sea were registered in Italy, compared to 69% in 2017 (UNHCR).

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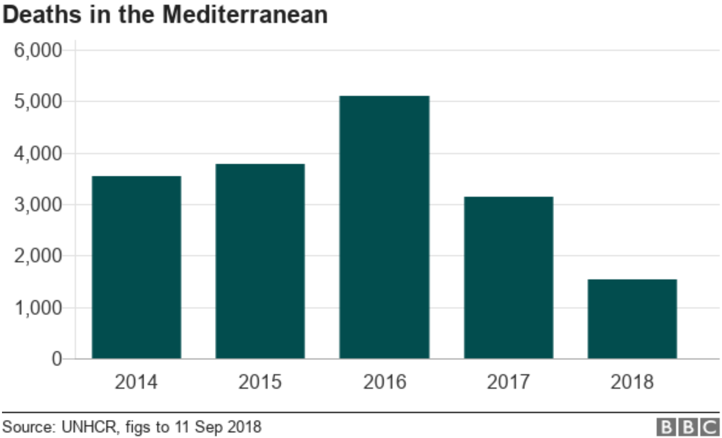

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- Kalashnykov and Kalashnykova, 2006: equivalence of the equilibrium to a solution of an appropriate variational inequality problem.

Initial work

G. Cappello, P. Daniele, *A Variational Formulation for a Human Migration Problem*, in AIRO Springer Series, 2019.

The mathematical model

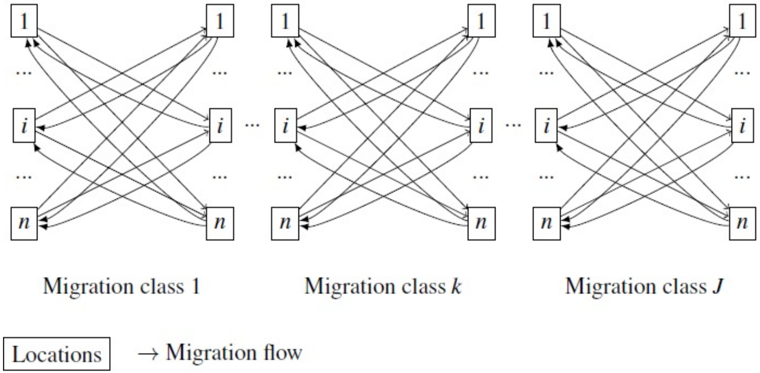

Figure: Multiclass migration network

Functions, parameters and decisions variables of the model

Symbol	Definition
$v_j^k(p^k)$	Destination utility function of any location j as percieved by the class k
$u_i^k(p^k)$	Origin utility function of any location i as percieved by the class k
$\bar{p}_i^k$	Initial population of class k in location i
p_i^k	Population at location i of class k
$c_{ij}^k(f)$	Movement cost from i to j for the class k
$c_{ji}^k(g)$	Movement cost from j to i for the class k
f_{ij}^k	Migration flow from i to j of class k
g_{ji}^k	Migration flow from j to i of class k
$w_{ij}^{k+}(p, f, g)$	Influence coefficient taken in account by an individual of class k moving from i to j
$w_{ij}^{k-}(p, f, g)$	Influence coefficient conjectured by an individual of class k moving from i to j

The mathematical model

Conservation of flow equations

$$p_i^k = \bar{p}_i^k - \sum_{\substack{j=1 \\ j \neq i}}^{n} f_{ij}^k + \sum_{\substack{j=1 \\ j \neq i}}^{n} g_{ji}^k \quad \forall i = 1, ..., n, \quad \forall k = 1, \ldots, J. \qquad (1)$$

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After the potential movement of migrants from location i to location j, each person of class k expects:

- a variation of the utility function value at j:

$$g_{ji}^k w_{ji}^{k+}(p^k, f^k, g^k) \frac{\partial v_j^k(p^k)}{\partial p_j^k},$$

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- a variation of the utility function value at i:

$$f_{ij}^k w_{ij}^{k-}(p^k, f^k, g^k) \frac{\partial u_i^k(p^k)}{\partial p_i^k}.$$

The optimization model

Social Optimum

$$\max_{(p^k, f^k, g^k) \in \mathbb{K}^k} U^k(p^k, f^k, g^k)$$

$$= \max_{(p^k, f^k, g^k) \in \mathbb{K}^k} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \Bigg(u_i^k(p^k) - f_{ij}^k w_{ij}^{k-}(p^k, f^k, g^k) \frac{\partial u_i^k(p^k)}{\partial p_i^k}$$

$$- c_{ij}^k(f^k) - c_{ji}^k(g^k) - v_j^k(p^k) + g_{ji}^k w_{ji}^{k+}(p^k, f^k, g^k) \frac{\partial v_j^k(p^k)}{\partial p_j^k} \Bigg)$$

where

$$\mathbb{K}^k = \Bigg\{ (p^k, f^k, g^k) \in \mathbb{R}_+^{n+2n(n-1)} : \; p_i^k = \bar{p}_i^k - \sum_{\substack{j=1 \\ j \neq i}}^{n} f_{ij}^k + \sum_{\substack{j=1 \\ j \neq i}}^{n} g_{ji}^k, \; \forall i;$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} f_{ij}^k \leq \bar{p}_i^k, \; \forall i, \; \sum_{\substack{i=1 \\ i \neq j}}^{n} g_{ji}^k \leq \bar{p}_j^k, \; \forall j \Bigg\}.$$

Variational formulation

Theorem

$$(p^*, f^*, g^*) \in \mathbb{K} = \prod_{k=1}^{J} \mathbb{K}^k \textit{ solution} \Leftrightarrow$$

$$\sum_{l=1}^{n} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{k=1}^{J} \Bigg(\frac{\partial v_j^k(p^{k*})}{\partial p_l^k} - g_{ji}^k \frac{\partial w_{ji}^{k+}(p^{k*}, f^{k*}, g^{k*})}{\partial p_l^k} \frac{\partial v_j^k(p^{k*})}{\partial p_j^k}$$

$$- g_{ji}^k w_{ji}^{k+}(p^{k*}, f^{k*}, g^{k*}) \frac{\partial^2 v_j^k(p^{k*})}{\partial p_j^k \partial p_l^k} - \frac{\partial u_i^k(p^{k*})}{\partial p_l^k}$$

$$+ f_{ij}^k \frac{\partial w_{ij}^{k-}(p^{k*}, f^{k*}, g^{k*})}{\partial p_l^k} \frac{\partial u_i^k(p^{k*})}{\partial p_i^k} + f_{ij}^k w_{ij}^{k-}(p^{k*}, f^{k*}, g^{k*}) \frac{\partial^2 u_i^k}{\partial p_i^k \partial p_l^k} \Bigg)$$

$$(p_l^k - p_l^{k*})$$

Theorem

$$
+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{J}\Bigg(-g_{ji}^{k*}\frac{\partial w_{ji}^{k+}(p^{k*},f^{k*},g^{k*})}{\partial f_{ij}^{k}}\frac{\partial v_j^k(p^{k*})}{\partial p_j^k}+\frac{\partial c_{ij}^k(f^{k*})}{\partial f_{ij}^k}
$$

$$
+w_{ij}^{k-}(p^{k*},f^{k*},g^{k*})\frac{\partial u_i^k(p^*)}{\partial p_i^k}+f_{ij}^{k*}\frac{\partial w_{ij}^{k-}(p^{k*},f^{k*},g^{k*})}{\partial f_{ij}^k}\frac{\partial u_i^k(p^{k*})}{\partial p_i^k}\Bigg)
$$

$$
(f_{ij}^k-f_{ij}^{k*})
$$

$$
+\sum_{j=1}^{n}\sum_{\substack{i=1\\ i\neq j}}^{n}\sum_{k=1}^{J}\Bigg(-w_{ji}^{k+}(p^{k*},f^{k*},g^{k*})\frac{\partial v_j^k(p^{k*})}{\partial p_j^k}
$$

$$
-g_{ji}^{k*}\frac{\partial w_{ji}^{k+}(p^{k*},f^{k*},g^{k*})}{\partial g_{ji}^k}\frac{\partial v_j^k(p^{k*})}{\partial p_j^k}+\frac{\partial c_{ji}^k(g^{k*})}{\partial g_{ji}^k}
$$

$$
+f_{ij}^{k*}\frac{\partial w_{ij}^{k-}(p^{k*},f^{k*},g^{k*})}{\partial g_{ji}^k}\frac{\partial u_i^k(p^{k*})}{\partial p_i^k}\Bigg)(g_{ji}^k-g_{ji}^{k*})\geq 0.
$$

Minimum problem

Variational inequality can be rewritten as a minimization problem:

$$V(p,f,g)=\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{J}\frac{\partial U^k(p^*,f^*,g^*)}{\partial p_j^k}(p_j^k-p_j^{k*})$$

$$+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{J}\frac{\partial U^k(p^*,f^*,g^{k*})}{\partial f_{ij}^k}(f_{ij}^k-f_{ij}^{k*})$$

$$+\sum_{j=1}^{n}\sum_{\substack{i=1\\ i\neq j}}^{n}\sum_{k=1}^{J}\frac{\partial U^k(p^*,f^*,g^{k*})}{\partial g_{ji}^k}(g_{ji}^k-g_{ji}^{k*}),$$

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$$+ \sum_{i=1}^{n} \sum_{\substack{j=1\\ j\neq i}}^{n} \sum_{k=1}^{J} \frac{\partial U^k(p^*, f^*, g^{k*})}{\partial f_{ij}^k}(f_{ij}^k - f_{ij}^{k*})$$

$$+ \sum_{j=1}^{n} \sum_{\substack{i=1\\ i\neq j}}^{n} \sum_{k=1}^{J} \frac{\partial U^k(p^*, f^*, g^{k*})}{\partial g_{ji}^k}(g_{ji}^k - g_{ji}^{k*}),$$

hence:

$$V(p,f,g) \geq 0 \text{ in } \mathbb{K} \text{ and } \min_{\mathbb{K}} V(p,f,g) = V(p^*, f^*, g^*) = 0.$$

Lagrange Function

$$\mathcal{L}(p, f, g, \lambda^1, \lambda^2, \lambda^3, \mu^1, \mu^2, \mu^3) = V(p, f, g)$$
$$+ \sum_{i=1}^{n} \sum_{k=1}^{J} \lambda_i^{k1}(-p_i^k) + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{k=1}^{J} \lambda_{ij}^{k2}(-f_{ij}^k)$$
$$+ \sum_{j=1}^{n} \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{k=1}^{J} \lambda_{ji}^{k3}(-g_{ji}^k) + \sum_{i=1}^{n} \sum_{k=1}^{J} \mu_{ik}^1 \left(p_i^k - \bar{p}_i^k + \sum_{\substack{j=1 \\ j \neq i}}^{n} f_{ij}^k + \sum_{\substack{j=1 \\ j \neq i}}^{n} g_{ji}^k \right)$$
$$+ \sum_{i=1}^{n} \sum_{k=1}^{J} \mu_{ik}^2 \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} f_{ij}^k - \bar{p}_i^k \right) + \sum_{j=1}^{n} \sum_{k=1}^{J} \mu_{jk}^3 \left(\sum_{\substack{i=1 \\ i \neq j}}^{n} g_{ji}^k - \bar{p}_i^k \right)$$

Existence of Lagrange Multipliers and Strong Duality

Theorem

The Lagrange multipliers do exist and, for all $i, j = 1, \ldots, n$, $i \neq j$ and $k = 1, \ldots, J$, the following conditions hold true:

$$\overline{\lambda}_i^{k1}(-p_i^{k*}) = 0, \quad \overline{\lambda}_{ij}^{k2}(-f_{ij}^{k*}) = 0, \quad \overline{\lambda}_{ji}^{k3}(-g_{ji}^{k*}) = 0, \tag{2}$$

$$\overline{\mu}_{ik}^2 \left(\sum_{\substack{j=1 \\ j\neq i}}^n f_{ij}^{k*} - \bar{p}_i^k \right) = 0, \quad \overline{\mu}_{jk}^3 \left(\sum_{\substack{i=1 \\ i\neq j}}^n g_{ji}^{k*} - \bar{p}_j^k \right) = 0 \tag{3}$$

$$\frac{\partial U^k(p^*, f^*, g^*)}{\partial p_j^k} - \overline{\lambda}_i^{k1} + \overline{\mu}_{ik}^1 = 0, \tag{4}$$

$$\frac{\partial U^k(p^*, f^*, g^*)}{\partial f_{ij}^k} - \overline{\lambda}_{ij}^{k2} + \overline{\mu}_{ik}^1 + \overline{\mu}_{ik}^2 = 0. \tag{5}$$

$$\frac{\partial U^k(p^*, f^*, g^*)}{\partial g_{ji}^k} - \overline{\lambda}_{ji}^{k3} + \overline{\mu}_{ik}^1 + \overline{\mu}_{jk}^3 = 0. \tag{6}$$

Existence of Lagrange Multipliers and Strong Duality

Theorem

Moreover, the strong duality also holds true, namely:

$$V(p^*, f^*, g^*) = \min_{\mathbb{K}} V(p, f, g)$$

$$= \max_{\substack{\lambda^1 \in \mathbb{R}^{Jn}_+, \lambda^2, \lambda^3 \in \mathbb{R}^{Jn(n-1)}_+ \\ \mu^1 \in \mathbb{R}^{nJ}, \mu^2, \mu^3 \in \mathbb{R}^{nJ}_+}} \min_{(p,f,g) \in \mathbb{R}^{Jn+2Jn(n-1)}} \mathcal{L}(p, f, g, \lambda^1, \lambda^2, \lambda^3, \mu^1, \mu^2, \mu^3).$$

Analysis of the expected net utilities with respect to the population at location i

$$\frac{\partial U^k(p^*, f^*, g^*)}{\partial p_j^k} - \overline{\lambda}_i^{k1} + \overline{\mu}_{ik}^1 = 0, \quad \forall i, \; i \neq j \text{ and } \forall k$$

$$\overset{p_i^{k*}>0}{\Longrightarrow} \frac{\partial U^k(p^*, f^*, g^*)}{\partial p_j^k} = -\overline{\mu}_{ik}^1, \quad \forall i, \; j, \; i \neq j \text{ and } k,$$

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Analysis of the expected net utilities with respect to the flow of population from j to i

$$\frac{\partial U^k(p^*, f^*, g^*)}{\partial f_{ij}^k} = \overline{\lambda}_{ij}^{k2} - \overline{\mu}_{ik}^1 - \overline{\mu}_{ik}^2, \quad \forall i,\ j,\ i \neq j \text{ and } k$$

$$\underset{f_{ij}^{k*}>0,\ \sum_{j=1, j\neq i}^{n} f_{ij}^{k*} = \overline{p}_i^k}{\Rightarrow} \quad \frac{\partial U^k(p^*, f^*, g^*)}{\partial f_{ij}^k} = -\overline{\mu}_{ik}^1, \quad \forall i,\ j,\ i \neq j \text{ and } k$$

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$$\overset{\overline{\lambda}_{ij}^{k2}>0,\overline{p}_i^k>0}{\Longrightarrow} \ \frac{\partial U^k(p^*, f^*, g^*)}{\partial f_{ij}^k} = \overline{\lambda}_{ij}^{k2} - \overline{\mu}_{ik}^1, \quad \forall i,\ j, n,\ i \neq j \text{ and } k$$

$$\overset{\overline{\mu}_{ik}^2>0}{\Rightarrow} \frac{\partial U^k(p^*, f^*, g^*)}{\partial f_{ij}^k} = -\overline{\mu}_{ik}^1 - \overline{\mu}_{ik}^2, \quad \forall i,\ j,\ i \neq j \text{ and } k$$

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In conclusion, we highlight that all the Lagrange variables give a precise evaluation of the migration phenomenon

Numerical Examples

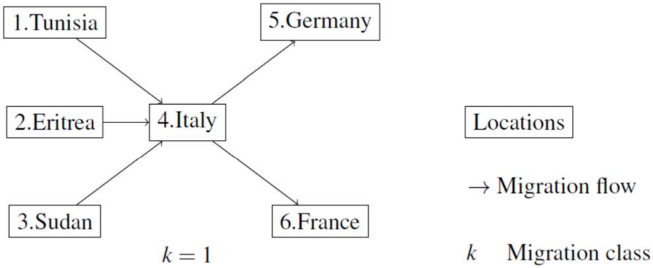

Fig. 2: **Multiclass migration network for the first numerical example**

Numerical Examples

Origin utility function	$u_1 = -17.7(p_1^1)^2$
	$u_2 = -0.9(p_2^1)^2$
	$u_3 = -4.04(p_3^1)^2$
	$u_4 = -27.8(p_4^1)^2$
	Source: *https://data.worldbank.org/*
Destination utility function	$v_4 = 6.31(p_4^1)^2$
	$v_5 = 1.9(p_5^1)^2$
	$v_6 = 5.16(p_6^1)^2$
	Source: *https://tradingeconomics.com/* *https://www.statista.com/*

Numerical Examples

Movement cost	$c_{14} = 3166.5 f^1_{14}$
	$c_{24} = 1250 f^1_{24}$
	$c_{34} = 350 f^1_{34}$
	$c_{45} = 350 f^1_{45}$
	$c_{46} = 350 f^1_{46}$.
	$c_{41} = 3166.5 g^1_{41}$
	$c_{42} = 1250 g^1_{42}$
	$c_{43} = 350 g^1_{43}$
	$c_{54} = 350 g^1_{54}$
	$c_{64} = 350 g^1_{64}$.
	Source: *https://www.mindsglobalspotlight.com*

Numerical Examples

Initial population

$$\bar{p}_1 = 4846976$$
$$\bar{p}_2^1 = 38647803$$
$$\bar{p}_3^1 = 11273661$$
$$\bar{p}_4^1 = 1036653$$
$$\bar{p}_5^1 = 740000$$
$$\bar{p}_6^1 = 2692236$$

Source: *https://www.worldometers.info/; ISTAT; INEED.*

Conservation of flow equations

$$p_1 = \bar{p}_1{}^1 - f_{14}$$
$$p_2 = \bar{p}_2{}^1 - f_{24}^1$$
$$p_3 = \bar{p}_3{}^1 - f_{34}^1$$
$$p_4 = \bar{p}_4{}^1 + g_{41}^1 + g_{42}^1 + g_{43}^1 - f_{45}^1 - f_{46}^1$$
$$p_5 = \bar{p}_5{}^1 - g_{45}^1$$
$$p_6 = \bar{p}_6{}^1 - g_{46}^1$$

Optimal Solutions

Year	Influence coefficients	Optimal flows
2015	$w_{14}^- = 0.8;\ w_{24}^- = 0.8$	$f_{14} = 25642711,16$
	$w_{34}^- = 0.8;\ w_{45}^- = 0.5$	$f_{24} = 0$
	$w_{46}^- = 0.5\ w_{14}^+ = 0.1$	$f_{34} = 0$
	$w_{24}^+ = 0.1\ w_{34}^+ = 0.1$	$f_{45} = 0$
	$w_{45}^+ = 0.1\ w_{46}^+ = 0.1$	$f_{46} = 1036653,232$
2016	$w_{14}^- = 0.9;\ w_{24}^- = 0.9$	$f_{14} = 9716195,819$
	$w_{34}^- = 0.9;\ w_{45}^- = 0.3$	$f_{24} = 10732881,68$
	$w_{46}^- = 0.3\ w_{14}^+ = 0.1$	$f_{34} = 0$
	$w_{24}^+ = 0.1\ w_{34}^+ = 0.1$	$f_{45} = 5987306,587$
	$w_{45}^+ = 0.1\ w_{46}^+ = 0.1$	$f_{46} = 19655404,58$

Optimal Solutions

Year	Influence coefficients	Optimal flows
2017	$w_{14}^{-}=1;\ w_{24}^{-}=1$	$f_{14}=23384293,81$
	$w_{34}^{-}=1;\ w_{45}^{-}=0.5$	$f_{24}=5516191,532$
	$w_{46}^{-}=0.5\ w_{14}^{+}=0.2$	$f_{34}=0$
	$w_{24}^{+}=0.25\ w_{34}^{+}=0.25$	$f_{45}=0$
	$w_{45}^{+}=0.2\ w_{46}^{+}=0.2$	$f_{46}=10732881,68$
2018	$w_{14}^{-}=1;\ w_{24}^{-}=1$	$f_{14}=34117175,49$
	$w_{34}^{-}=1;\ w_{45}^{-}=0.6$	$f_{24}=3026572,161$
	$w_{46}^{-}=0.6\ w_{14}^{+}=0.4$	$f_{34}=0$
	$w_{24}^{+}=0.3\ w_{34}^{+}=0.3$	$f_{45}=0$
	$w_{45}^{+}=0.3\ w_{46}^{+}=0.3$	$f_{46}=5516191,068$

Analysis of the results

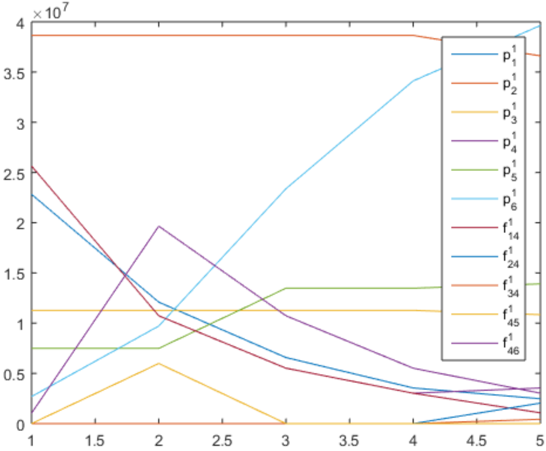

Figure: **Optimal flows and populations**

Future research

Extended models

- Comparison between the social and the migrant's point of view

Future research

Extended models

- Comparison between the social and the migrant's point of view
- Introduction of governmental regulations