Innovative Strategies for Sustainable and Efficient Last-Mile Delivery

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Joint work with G. Colajanni - D. Sciacca

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Ministero dell'Università e della Ricerca

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PIANO NAZIONALE DI RIPRESA E RESILIENZA

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- University of Naples Federico II
- Institute of System Analysis and Computer Science "Antonio Ruberti" of the National Research Council (IASI-CNR)
- University of Catania

Outline

1. Introduction

Outline

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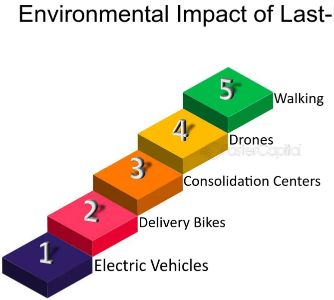
E-commerce

- The rapid growth of e-commerce has intensified the need for efficient last-mile delivery solutions

Environmental concerns

- Rise of environmental concerns and the emergence of new business models have prompted companies to explore alternative strategies to optimize last-mile delivery



Green Vehicles

- One promising approach is the integration of green vehicles into the last-mile delivery process

Crowdsoursing

- Another approach is the integration of crowdsourced drivers into the last-mile delivery process

Regular Vehicles and Occasional Drivers

- We develop a model to optimize last-mile delivery operations using a combination of RVs and ODs

Parcel Lockers

- Another approach is the use of parcel lockers which eliminate the inconvenience of missed deliveries and restrictive delivery schedules.

Truck-drone Systems

- Another solution is the truck-drone system which minimizes travel distances for trucks, allowing them to focus on major roads while drones handle short-distance deliveries.

Literature Overview

1. C. Archetti, M. Savelsbergh, M.G. Speranza, *The Vehicle Routing Problem with Occasional Drivers*, European Journal of Operational Research, 254, 2016, 472-480

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4. G. Colajanni, P. Daniele, A. Nagurney, *Centralized Supply Chain Network Optimization with UAV-based last mile deliveries*, Tranportation Research Part C, 155, 2023, 104316

The Mathematical Model with Crowdsourcing

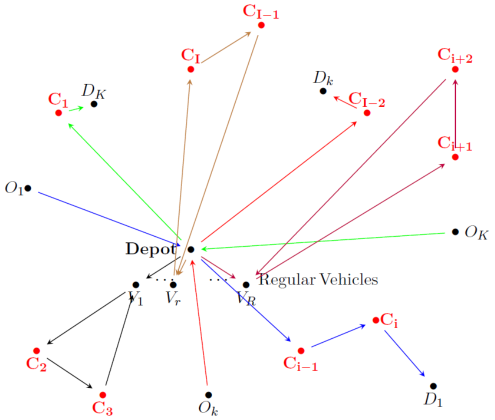

Figure: Network topology

The Mathematical Model with Crowdsourcing

Assumptions:

1. All customers' requests have to be satisfied

The Mathematical Model with Crowdsourcing

Assumptions:

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2. Both ODs and RVs can serve more than one customer

The Mathematical Model with Crowdsourcing

Assumptions:

1. All customers' requests have to be satisfied
2. Both ODs and RVs can serve more than one customer
3. Both ODs and RVs can use petrol/diesel vehicles or electric vehicles

The Supply Chain Network

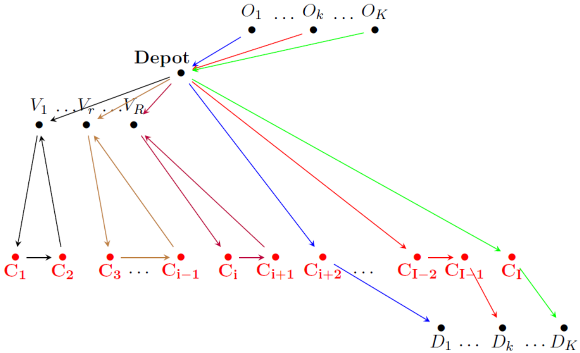

Figure: Supply chain representation

Outline

Company's behavior

Company's objective

$$\max \left\{ \sum_{i=1}^{I} \sum_{l=1}^{2} p_i^l \left[\sum_{r=1}^{R} x_{ri}^l + \sum_{k=1}^{K} y_{ki}^l \right] + N \sum_{i=1}^{I} \left[\sum_{r=1}^{R} x_{ri}^2 + \sum_{k=1}^{K} y_{ki}^2 \right] \right.$$

Company's behavior

Company's objective

$$
\begin{aligned}
\max \Bigg\{ & \sum_{i=1}^{I}\sum_{l=1}^{2} p_i^l \left[\sum_{r=1}^{R} x_{ri}^l + \sum_{k=1}^{K} y_{ki}^l\right] + N \sum_{i=1}^{I}\left[\sum_{r=1}^{R} x_{ri}^2 + \sum_{k=1}^{K} y_{ki}^2\right] \\
& -\gamma(x,y) - \sum_{r=1}^{R}\sum_{l=1}^{2}\left(c_r^{BI} + c_r^{MI} + c_r^{SI}(x_r^l)\right) - \sum_{r=1}^{R} c_r^D \\
& - \sum_{r=1}^{R}\sum_{i=1}^{I}\sum_{l=1}^{2} c_{ri}^l(x_{ri}^l) - \sum_{k=1}^{K} c^{OD} - \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{2} c_{ki}^l(y_{ki}^l)
\end{aligned}
$$

Company's behavior

Company's objective

$$
\begin{aligned}
\max \Bigg\{ & \sum_{i=1}^{I}\sum_{l=1}^{2} p_i^l \left[\sum_{r=1}^{R} x_{ri}^l + \sum_{k=1}^{K} y_{ki}^l\right] + N \sum_{i=1}^{I}\left[\sum_{r=1}^{R} x_{ri}^2 + \sum_{k=1}^{K} y_{ki}^2\right] \\
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& -\sum_{r=1}^{R}\sum_{i=1}^{I}\sum_{l=1}^{2} c_{ri}^l(x_{ri}^l) - \sum_{k=1}^{K} c^{OD} - \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{2} c_{ki}^l(y_{ki}^l) \\
& -\sum_{i=1}^{I}\sum_{l=1}^{2} P_i^l(\rho_i^l)\left[\sum_{r=1}^{R} d_{ri}^l + \sum_{k=1}^{K} d_{ki}^l\right] \Bigg\}
\end{aligned}
$$

Company's behavior

Constraints

- Maximum capacity of each RV (per vehicle) of the company:

$$\sum_{i=1}^{I} x_{ri}^{l} \leq \overline{C}_{r}^{l} \quad \forall r = 1, \ldots, R, \ \forall l = 1, 2$$

Company's behavior

Constraints

- Maximum capacity of each RV (per vehicle) of the company:

$$\sum_{i=1}^{I} x_{ri}^{l} \leq \overline{C}_r^l \quad \forall r = 1, \dots, R, \ \forall l = 1, 2$$

- Delay (given by the difference between the sum of the delivery times to the customers preceding i and the time within which to make the delivery to i):

$$d_{ri}^{l} \geq \sum_{j=1}^{I} \delta_{ji}^{rl} t_{rj}^{l}(x_r^l) - \overline{T}_i, \quad \forall r = 1, \dots, R, \ \forall i = 1, \dots, I, \ \forall l = 1, 2$$

$$d_{ri}^{l} \geq 0, \quad \forall r = 1, \dots, R, \ \forall i = 1, \dots, I, \ \forall l = 1, 2$$

Outline

ODs' behavior

ODs' objective

$$\max \left\{ \sum_{i=1}^{I} \sum_{l=1}^{2} c_{ki}^{l}(y_{ki}^{l}) - \sum_{l=1}^{2} \tilde{c}_k^{MI} - \sum_{l=1}^{2} \tilde{c}_k^{SI}(y_k^l) - \sum_{i=1}^{I} \sum_{l=1}^{2} \tilde{c}_{ki}^{l}(y_{ki}^{l}) \right.$$

ODs' behavior

ODs' objective

$$\max\left\{\sum_{i=1}^{I}\sum_{l=1}^{2} c_{ki}^{l}(y_{ki}^{l}) - \sum_{l=1}^{2} \tilde{c}_{k}^{Ml} - \sum_{l=1}^{2} \tilde{c}_{k}^{Sl}(y_{k}^{l}) - \sum_{i=1}^{I}\sum_{l=1}^{2} \tilde{c}_{ki}^{l}(y_{ki}^{l}) \right.$$
$$\left. -\alpha_k \sum_{i=1}^{I} V_i d_{ki} \right\}$$

ODs' behavior

Constraints

- Maximum capacity of OD k, at time t (for both types of vehicles):

$$\sum_{i=1}^{I} y_{ki}^{l} \leq \overline{C}_{k}^{l} \quad \forall l = 1, 2$$

ODs' behavior

Constraints

- Maximum capacity of OD k, at time t (for both types of vehicles):

$$\sum_{i=1}^{I} y_{ki}^{l} \leq \overline{C}_{k}^{l} \quad \forall l = 1, 2$$

- Availability of OD k to visit a customer with a vehicle of type l:

$$y_{ki}^{l} \leq M\beta_{ki}^{l} \quad \forall i = 1, \ldots, I, \; \forall l = 1, 2$$

ODs' behavior

Constraints

- Maximum capacity of OD k, at time t (for both types of vehicles):

$$\sum_{i=1}^{I} y_{ki}^{l} \leq \overline{C}_{k}^{l} \quad \forall l = 1, 2$$

- Availability of OD k to visit a customer with a vehicle of type l:

$$y_{ki}^{l} \leq M\beta_{ki}^{l} \quad \forall i = 1, \dots, I, \ \forall l = 1, 2$$

- Delay (given by the difference between the total delivery times and the time within which delivery to i must be made):

$$d_{ki}^{l} \geq \sum_{j=1}^{I} \delta_{ji}^{kl} t_{kj}^{l}(y_{k}^{l}) - \overline{T}_{i}, \quad \forall i = 1, \dots, I, \ \forall l = 1, 2.$$

ODs' behavior

Constraints

- Maximum time available for k, with vehicle l:

$$\sum_{i=1}^{I} t_{ki}^{l}(y_k^l) \le \overline{T}_k^l, \quad \forall l = 1, 2.$$

Outline

Customers' behavior

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Customers' equilibrium conditions

$$p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^l) d_{ri}^l \begin{cases} = \rho_i^{l*} & \text{if } x_{ri}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } x_{ri}^{l*} = 0 \end{cases} \quad \forall r = 1, \ldots, R, \ \forall l = 1, 2$$

Customers' behavior

Customers' equilibrium conditions

$$p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^l) d_{ri}^l \begin{cases} = \rho_i^{l*} & \text{if } x_{ri}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } x_{ri}^{l*} = 0 \end{cases} \quad \forall r = 1, \dots, R, \ \forall l = 1, 2$$

$$p_i^l + \hat{c}_{ki}^l(y_{ki}^{l*}) - P_i^l(\rho_i^l) d_{ki}^l \begin{cases} = \rho_i^{l*} & \text{if } y_{ki}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } y_{ki}^{l*} = 0 \end{cases} \quad \forall k = 1, \dots, K, \ \forall l = 1, 2$$

Customers' behavior

Customers' equilibrium conditions

$$p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^l) d_{ri}^l \begin{cases} = \rho_i^{l*} & \text{if } x_{ri}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } x_{ri}^{l*} = 0 \end{cases} \quad \forall r = 1, \dots, R, \ \forall l = 1, 2$$

$$p_i^l + \hat{c}_{ki}^l(y_{ki}^{l*}) - P_i^l(\rho_i^l) d_{ki}^l \begin{cases} = \rho_i^{l*} & \text{if } y_{ki}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } y_{ki}^{l*} = 0 \end{cases} \quad \forall k = 1, \dots, K, \ \forall l = 1, 2$$

$$\phi_i^l = \sum_{r=1}^{R} x_{ri}^{l*} + \sum_{k=1}^{K} y_{ki}^{l*} \quad \forall i = 1, \dots, I, \ \forall l = 1, 2$$

Outline

Company's VI

Company's VI

Define:

$$
\begin{aligned}
f^1(X^1) = & \sum_{i=1}^{I}\sum_{l=1}^{2} p_i^l \left[\sum_{r=1}^{R} x_{ri}^l + \sum_{k=1}^{K} y_{ki}^l\right] + N\sum_{i=1}^{I}\left[\sum_{r=1}^{R} x_{ri}^2 + \sum_{k=1}^{K} y_{ki}^2\right] \\
& - \gamma(x,y) - \sum_{r=1}^{R}\sum_{l=1}^{2}\left(c_r^{Bl} + c_r^{Ml} + c_r^{Sl}(x_r^l)\right) - \sum_{r=1}^{R} c_r^D \\
& - \sum_{r=1}^{R}\sum_{i=1}^{I}\sum_{l=1}^{2} c_{ri}^l(x_{ri}^l) - \sum_{k=1}^{K} c^{OD} \\
& - \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{2} c_{ki}^l(y_{ki}^l) - \sum_{i=1}^{I} P_i^l(\rho_i^l)\left[\sum_{r=1}^{R} d_{ri} + \sum_{k=1}^{K} d_{ki}\right],
\end{aligned}
$$

where $X^1 = (x, d^1) \in \; K^1 \subseteq \mathbb{R}^{IR(L+1)}$

Company's VI

Theorem

A vector $X^{1*} \in K^1$ is an optimal solution to the Delivery Company Optimization Problem if and only if is a solution to the Variational Inequality:
Find $X^{1*} \in K^1$ such that:

$$\langle \nabla f^1(X^{1*}), X^1 - X^{1*} \rangle \geq 0, \quad \forall X^1 \in K^1$$

Moreover, the variational inequality admits at least one solution.

Company's VI

Theorem

Suppose that, for each costumer i, for each regular vehicle r, for each occasional driver k and for each mean of transport l, the cost functions c_r^{Sl}, c_{ri}^l, c_{ki}^l, P_i^l constitute families of strictly monotone functions w.r.t. their own variables. Moreover, suppose that, the depot management cost function γ is additive, that is:

$$\gamma(x, y) = \gamma^1(x) + \gamma^2(y)$$

and that γ is a strictly convex function. Then, the operator $F^1(X^1) = \nabla f^1(X^1)$ is a strictly monotone function and the solution to the variational inequality is unique.

ODs' VI

ODs' VI

Define:

$$
\begin{aligned}
f_k^2(X_k^2) \quad = \quad & \sum_{i=1}^{I}\sum_{l=1}^{2} c_{ki}^l(y_{ki}^l) - \sum_{l=1}^{2} \tilde{c}_k^{Ml} - \sum_{l=1}^{2} \tilde{c}_k^{Sl}(y_k^l) \\
& - \sum_{i=1}^{I}\sum_{l=1}^{2} \tilde{c}_{ki}^l(y_{ki}^l) - \alpha_k \sum_{i=1}^{I} V_i d_{ki}
\end{aligned}
$$

where $X_k^2 = (y_k, d_k^2) \in K_k^2 \subseteq \mathbb{R}^{3I}$

ODs' VI

Theorem

A vector $X^{2*} = (X_k^{2*})_{k=1,\dots,K} \in \mathcal{K}^2 = \prod_{k=1}^{K} \mathcal{K}_k^2 \subseteq \mathbb{R}^{3KI}$ *is an optimal solution to the optimization problems if and only if is a solution to the variational inequality:*
Find $X^{2*} \in \mathcal{K}^2$ *such that:*

$$\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{2}\left[\frac{\partial \tilde{c}_{ki}^{l}(y_{ki}^{l*})}{\partial y_{ki}^{l}} + \frac{\partial \tilde{c}_{k}^{Sl}(y_{k}^{l*})}{\partial y_{ki}^{l}} - \frac{\partial c_{ki}^{l}(y_{ki}^{l*})}{\partial y_{ki}^{l}} - p_i^l - \delta_l N\right]$$
$$\times (y_{ki}^{l} - y_{ki}^{l*}) + \sum_{k=1}^{K}\sum_{i=1}^{I}[\alpha_k V_i] \times (d_{ki} - d_{ki}^{*}) \geq 0, \quad \forall X^2 \in \mathcal{K}^2,$$

where $\delta_l = 1$ *if* $l = 2$, $\delta_l = 0$ *otherwise. Moreover, such a variational inequality admits at least one solution.*

Customers' VI

Customers' VI

Define:

$$X^3 = (x, y, \rho) \in K^3 := \mathbb{R}^{I(RL+KL+1)}$$

Customers' VI

Theorem

A vector $X^{3*} \in K^3$ satisfies the equilibrium conditions if and only it is a solution to the variational inequality:
Find $X^{3*} \in K^3$ such that:

$$\sum_{r=1}^{R}\sum_{i=1}^{I}\sum_{l=1}^{2}\left[p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^{l*})d_{ri}^{l*} - \rho_i^{l*}\right] \times (x_{ri}^l - x_{ri}^{l*})$$
$$+\sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{2}\left[p_i^l + \hat{c}_{ki}^l(y_{ki}^{l*}) - P_i^l(\rho_i^{l*})d_{ki}^{l*} - \rho_i^{l*}\right] \times (y_{ki}^l - y_{ki}^{l*})$$
$$+\sum_{i=1}^{I}\left[\sum_{r=1}^{R} x_{ri}^{l*} + \sum_{k=1}^{K} y_{ki}^{l*} - \phi_i^l(\rho_i^{l*})\right] \times (\rho_i^l - \rho_i^{l*}) \geq 0, \quad \forall X^3 \in K^3$$

Moreover, such a variational inequality admits at least one solution.

VI for the entire Network

Definition (Network Equilibrium)

An equilibrium state of the presented network is reached if the flows between the tiers of the network coincide and the product shipments and prices satisfy the sum of the optimality conditions.

VI for the entire Network

Theorem

A vector $X^* = (X^{1*}, X^{2*}, X^{3*}) \in \mathcal{K}$ is an equilibrium state of the network if and only if it is a solution to the VI given by the sum of the previous VIs, where the feasible set is

$$\mathcal{K} := \left\{ X \in \mathbb{R}^{I(RL+R+KL+K+1)} : \text{ all the constraints are satisfied} \right\}.$$

VI for the entire Network

Theorem

A vector $X^* = (X^{1*}, X^{2*}, X^{3*}) \in \mathcal{K}$ is an equilibrium state of the network if and only if it is a solution to the VI given by the sum of the previous VIs, where the feasible set is

$$\mathcal{K} := \left\{ X \in \mathbb{R}^{I(RL+R+KL+K+1)} : \ \textit{all the constraints are satisfied} \right\}.$$

Theorem

Under the previous assumptions, the solution to the total VI is unique.

Model with Parcel Lockers

Objective:

determine the optimal allocation of lockers in order to minimize the total cost of the parcel delivery company

$$\min \sum_{i=1}^{n} [F_i x_i + G_i \cdot (x_i + l_i - y_i) + D_i y_i + Q_i C_i (1 - \rho_i) - R_i Q_i \rho_i]$$

$$x_i, y_i, z_{ik} \in \{0, 1\}, \quad \rho_i \in [0, 1] \quad \forall i, k = 1, \dots, n. \tag{1}$$

$$x_i \leq (1 - l_i), \quad \forall i = 1, \dots, n; \tag{2}$$

$$y_i \leq l_i, \quad \forall i = 1, \dots, n; \tag{3}$$

Model with Parcel Lockers

$$\rho_i \geq p_{ik} \cdot (l_k + x_k - y_k), \quad \forall i, k = 1, \dots, n; \tag{4}$$

$$\rho_i \leq (1 - z_{ik}) + [p_{ik} \cdot (l_k + x_k - y_k)], \quad \forall i, k = 1, \dots, n; \tag{5}$$

$$\sum_{k=1}^{n} z_{ik} \geq 1, \quad \forall i = 1, \dots, n; \tag{6}$$

Remark:

Constraints (4)-(6) are equivalent to the following nonlinear constraint:

$$\rho_i = \max\{p_{ik}(d_{ik}) \cdot (l_k + x_k - y_k)\},$$

that is, the value of ρ_i equals the maximum percentage of customers who go to the nearest open locker

Model with Truck-drone systems

Objectives:

- determine which nodes/customers are visited by each drone (and in which launch) and which are visited by the truck;

Model with Truck-drone systems

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- determine which nodes/customers are visited by each drone (and in which launch) and which are visited by the truck;
- identify the specific kilometer at which the drone is launched and retrieved;

Model with Truck-drone systems

Objectives:

- determine which nodes/customers are visited by each drone (and in which launch) and which are visited by the truck;
- identify the specific kilometer at which the drone is launched and retrieved;
- establish the exact route of the truck;

Model with Truck-drone systems

Objectives:

- determine which nodes/customers are visited by each drone (and in which launch) and which are visited by the truck;
- identify the specific kilometer at which the drone is launched and retrieved;
- establish the exact route of the truck;
- specify the kilometer at which the drone's battery replacement occurs;

Model with Truck-drone systems

Objectives:

- determine which nodes/customers are visited by each drone (and in which launch) and which are visited by the truck;
- identify the specific kilometer at which the drone is launched and retrieved;
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- specify the kilometer at which the drone's battery replacement occurs;

while

- minimizing costs and emissions;

Model with Truck-drone systems

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while

- minimizing costs and emissions;
- minimizing the penalty associated with delays;

Model with Truck-drone systems

Objectives:

- determine which nodes/customers are visited by each drone (and in which launch) and which are visited by the truck;
- identify the specific kilometer at which the drone is launched and retrieved;
- establish the exact route of the truck;
- specify the kilometer at which the drone's battery replacement occurs;

while

- minimizing costs and emissions;
- minimizing the penalty associated with delays;
- minimizing the waiting times (which incur costs) for the truck and drones during recovery.

Model with Truck-drone systems

$$
\begin{aligned}
\min \Bigg\{ & \sum_{i,j} \Bigg[t\left(p_{ij}^{T}\right) + \sum_{n,d} t\left(p_{ij}^{nd,(D)}\right) + \sum_{n,d,k} \left[t\left(d_{ijk}^{(L)}(x_{ijk}^{nd})\right) + t\left(d_{ijk}^{(R)}(y_{ijk}^{nd})\right)\right]\Bigg] + \sum_{n,d}\left[\gamma^{(T)} a_{nd}^{(T)} + \gamma^{(D)} a_{nd}^{(D)}\right] \\
& + \sum_{i,j}\Bigg[\alpha^{(T)}\left(p_{ij}^{(T)}\right) + \sum_{n,d}\alpha^{(D)}\left(p_{ij}^{nd,(D)}\right) + \sum_{n,d,k}\left[\alpha^{(D)}\left(d_{ijk}^{(L)}(x_{ijk}^{nd})\right) + \alpha^{(D)}\left(d_{ijk}^{(R)}(y_{ik}^{nd})\right)\right]\Bigg] + \sum_{i}\beta_i r_i \\
& + M\Bigg[\sum_k \sqrt{\left(\sum_{i,j,n,d}(x_{ijk}^{nd})^2\right)\left(\sum_{i,j} z_{ij}^2\right) + \epsilon} + \sqrt{\sum_{i,j,k,n,d}\left[(x_{ikj}^{nd})^2(1-\tilde{x}_{ijk}^{nd})^2\right] + \epsilon} \\
& + \sqrt{\sum_{i,j,k,n,d}\left[(y_{ikj}^{nd})^2(1-\tilde{y}_{ijk}^{nd})^2\right] + \epsilon} + \sum_k \sqrt{\left(\sum_{i,j,n,d}(y_{ijk}^{nd})^2\right)\left(\sum_i z_{ik}^2\right) + \epsilon} \\
& + \sum_k \sqrt{\left(\sum_{i,n,d}(w_{ik}^{nd})^2\right)\left(\sum_i z_{ik}^2\right) + \epsilon} + \sqrt{\sum_{i,j} z_{ij}^2 \cdot \tilde{z}_{ij}^2 + \epsilon} + \sqrt{\sum_{i,j,n,d}(w_{ij}^{nd})^2(\tilde{w}_{ij}^{nd})^2 + \epsilon}\Bigg]\Bigg\}
\end{aligned}
$$

$$
\sum_{ijk}(x_{ijk}^{nd} + y_{ijk}^{nd}) + \sum_i \left(\sum_{n,d} w_{ik}^{nd} + z_{ik}\right) \geq 0, \quad \forall i,j \tag{7}
$$

$$
(1-\tilde{z}_{ij})d_{ij}^{(T)} = p_{ij}^{(T)}, \quad \forall i,j \tag{8}
$$

$$
z_{ij}d_{ij}^{(T)} = p_{ij}^{(T)}, \quad \forall i,j \tag{9}
$$

Model with Truck-drone systems

$$w_{ij}^{nd} \cdot d_{ij}^{(D)} = p_{ij}^{nd,(D)}, \quad \forall i, j, n, d \tag{10}$$

$$(1 - \tilde{w}_{ij}^{nd}) \cdot d_{ij}^{(D)} = p_{ij}^{nd,(D)}, \quad \forall i, j, n, d \tag{11}$$

$$\sum_{i} z_{ij} = \sum_{k} z_{jk}, \quad \forall j \tag{12}$$

$$\sum_{i,k} \tilde{x}_{ikj}^{nd} + \sum_{i} w_{ij}^{nd} = \sum_{k} w_{jk}^{nd} + \sum_{i,k} \tilde{y}_{ikj}^{nd}, \quad \forall j, n, d \tag{13}$$

$$\tilde{x}_{ikj}^{nd} \in [0, 1], \quad \forall i, k, j \tag{14}$$

$$\tilde{x}_{ikj}^{nd} \leq x_{ikj}^{nd} \cdot G, \quad \forall i, k, j, n, d \tag{15}$$

$$\tilde{y}_{ikj}^{nd} \in [0, 1], \quad \forall i, k, j, n, d \tag{16}$$

$$\tilde{y}_{ikj}^{nd} \leq y_{ikj}^{nd} \cdot G, \quad \forall i, k, j, n, d \tag{17}$$

$$\sum_{i,j,k} x_{ijk}^{nd} D_k + \sum_{i,j} w_{ij}^{nd} D_j \leq C^d, \quad \forall n, d \tag{18}$$

Model with Truck-drone systems

$$\begin{aligned} a_{nd}^{(T)} \geq & \left[\sum_{ijk} t\left(d_{ijk}^{(L)}(x_{ijk}^{nd})\right) + \sum_{i,j} w_{ij}^{nd}\, t\left(p_{ij}^{nd,(D)}\right) + \sum_{i,j,k} t\left(d_{ijk}^{(R)}(y_{ijk}^{nd})\right)\right] \\ & - \left[\sum_{i,j,k}\left[\tilde{x}_{ijk}^{nd}\, t\left(p_{ij}^{(T)} - x_{ij}^{nd}\right) + \sum_{\hat{i},\hat{j},\hat{k}}\left[\tilde{y}_{\hat{i}\hat{j}\hat{k}}\, t\left(y_{\hat{i}\hat{j}\hat{k}} - p_{\hat{i}\hat{j}}^{(T)}\right) + \sum_{j\leq\tilde{i},\tilde{j}\leq\hat{i}} t\left(p_{\tilde{i}\tilde{j}}^{(T)}\right)\right.\right.\right. \\ & \left.\left.\left. + \sum_{\bar{d}\neq d}\sum_{\bar{n}}\sum_{j\leq\bar{i},\bar{j},\bar{k}\leq\hat{i}}\left[t_{\bar{n}\bar{d}}^{(L)}\tilde{x}_{\bar{i}\bar{j}\bar{k}}^{\bar{n}\bar{d}} + \left[t_{\bar{n}\bar{d}}^{(R)} + a_{\bar{n}\bar{d}}^{(R)}\right]\tilde{y}_{\bar{i}\bar{j}}^{\bar{n}\bar{d}}\right]\right]\right]\right], \quad \forall n, d \end{aligned} \tag{19}$$

$$\begin{aligned} & \sum_{k} z_{ki}\left[\sum_{j\leq i-1, l\leq i}\left[t\left(p_{jl}^{(T)}\right) + l_{\tilde{j}\tilde{l}}^{(T)} + \sum_{n,d,m}\left[t_{nd}^{(L)}\tilde{x}_{jlm}^{nd} + \left(t_{nd}^{(R)} + a_{nd}^{(T)}\tilde{y}_{jln}^{nd}\right)\right]\right]\right] \\ & + \sum_{n,d,j,l}\tilde{x}_{jli}^{nd}\left[\sum_{\bar{j}\leq j-1,\bar{l}\leq j}\left[t\left(p_{\bar{j}\bar{l}}^{(T)}\right) + l_{\bar{j}\bar{l}}^{(T)} + \sum_{\bar{n},\bar{d},n}\left(t_{\bar{n}\bar{l}}^{(L)}\tilde{x}_{\bar{j}\bar{l}n} + \left(t_{\bar{n}\bar{d}}^{(R)} + a_{\bar{n}\bar{d}}^{(T)}\right)\tilde{y}_{\bar{j}\bar{l}m}^{\bar{n}\bar{d}} + l_{\bar{n}\bar{d}}\right)\right]\right. \\ & \left. + t\left(x_{jli}^{nd}\right) + t_{nd}^{(L)} + t\left(d_{jli}^{(L)}(x_{jli}^{nd})\right)\right] \\ & + \sum_{n,d,j} w_{ji}^{nd}\left[\sum_{\bar{j}\leq i-1, l\leq i}\left[t\left(p_{\bar{j}\bar{l}}^{(T)}\right) + l_{\bar{j}\bar{l}}^{(T)} + \sum_{\bar{n},\bar{d},n}\left(t_{\bar{n}\bar{l}}^{(L)}\tilde{x}_{\bar{j}\bar{l}n} + \left(t_{\bar{n}\bar{d}}^{(R)} + a_{\bar{n}\bar{d}}^{(T)}\right)\tilde{y}_{\bar{j}\bar{l}m}^{\bar{n}\bar{d}} + l_{\bar{n}\bar{d}}\right)\right.\right. \\ & \left.\left. + \tilde{x}_{\bar{j}\bar{l}i}^{nd}\left[t(x_{\bar{j}\bar{l}i}^{nd}) + t_{nd}^{(L)} + t\left(d_{\bar{j}\bar{l}i}^{(L)}\left(x_{\bar{j}\bar{l}i}^{nd}\right)\right)\right] + t\left(p_{\bar{j}\bar{l}}^{nd,(D)} + l_{\bar{j}\bar{l}}^{nd}\right)\right]\right] - r_i \leq \bar{T}_i, \quad i \end{aligned} \tag{20}$$

Model with Truck-drone systems

$$a_{nd}^{(D)} \geq -a_{nd}^{(T)}, \quad \forall n, d \tag{21}$$

$$\sum_{i,j,k} \tilde{x}_{ijk}^{nd} \leq 1, \quad \forall n, d \tag{22}$$

$$\sum_{i,j,k} \tilde{x}_{ijk}^{nd} = \sum_{\overline{ijk}} \tilde{y}_{\overline{ijk}}^{nd}, \quad \forall n, d \tag{23}$$

$$\sum_{k} \tilde{x}_{ijk}^{nd} \leq z_{ij}, \quad \forall i, j, n, d \tag{24}$$

$$\sum_{k} \tilde{y}_{ijk}^{nd} \leq z_{ij}, \quad \forall i, j, n, d \tag{25}$$

$$\sum_{\overline{ijl}} \tilde{x}_{\overline{ijl}}^{nd} \leq \sum_{ijl} \tilde{y}_{ijl}^{(n-1)d}, \quad \forall n, d \tag{26}$$

A Numerical Simulation with Crowdsoursing

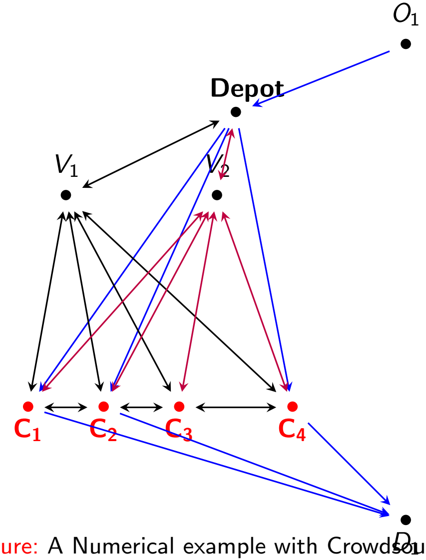

Figure: A Numerical example with Crowdsoursing

A Numerical Simulation with Crowdsoursing

University of Catania

Patrizia DANIELE

Data

- 2 RDs with a petrol/diesel vehicle and an electric vehicle
- $\overline{C}_1^1 = 100,\ \overline{C}_1^2 = 70,\ \overline{C}_2^1 = 150,\ \overline{C}_2^2 = 90$
- 1 occasional driver with an electric vehicle
- $\overline{C}_k^2 = 60$
- 4 customers
- $\beta_{13}^1 = \beta_{13}^2 = 0,\ \beta_{11}^l = \beta_{12}^l = \beta_{14}^l = 1$

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Two Simulations

S1: without occasional drivers;

S2: with occasional driver.

A Numerical Simulation with Crowdsoursing

S1	$i=1$	$i=2$	$i=3$	$i=4$	**Objective Function**
x_{1i}^{1*}	0	0	0	0	
x_{1i}^{2*}	18.84	8.69	24.52	17.95	
x_{2i}^{1*}	0	0	0	0	
x_{2i}^{2*}	21.16	11.31	15.48	12.05	
y_{1i}^{1*}	-	-	-	-	
y_{1i}^{2*}	-	-	-	-	11,728

Table: Optimal solutions in S1

A Numerical Simulation with Crowdsoursing

S1	$i=1$	$i=2$	$i=3$	$i=4$	**Objective Function**
x_{1i}^{1*}	0	0	0	0	
x_{1i}^{2*}	18.84	8.69	24.52	17.95	
x_{2i}^{1*}	0	0	0	0	
x_{2i}^{2*}	21.16	11.31	15.48	12.05	
y_{1i}^{1*}	-	-	-	-	
y_{1i}^{2*}	-	-	-	-	11,728

Table: Optimal solutions in S1

S2	$i=1$	$i=2$	$i=3$	$i=4$	**Objective Function**
x_{1i}^{1*}	0	0	0	0	
x_{1i}^{2*}	5.53	2.66	26.44	11.70	
x_{2i}^{1*}	0	0	0	0	
x_{2i}^{2*}	5.33	2.55	13.56	5.87	
y_{1i}^{1*}	-	-	-	-	
y_{1i}^{2*}	29.14	14.79	0	12.43	12,150

Table: Optimal solutions in S2

A Numerical Simulation with Crowdsourcing

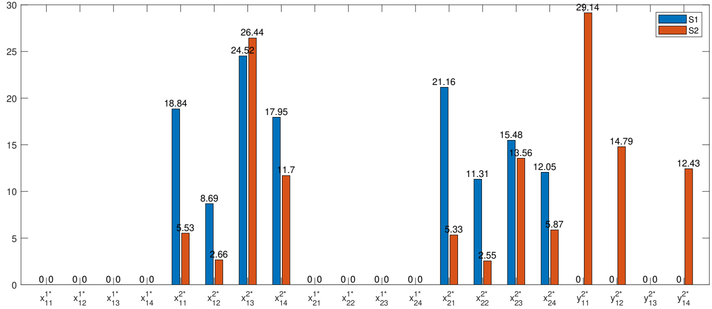

Figure: Comparison of optimal solutions in S1 and S2


THANK YOU
FOR YOUR
ATTENTION

Happy Birthday!