



OPTIMIZATION OF GREEN VEHICLES AND CROWDSORCING IN THE LAST MILE DELIVERIES

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JOINT PAPER WITH G. COLAJANNI - D. SCIACCA

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Outline



Ministero
dell'Università
e della Ricerca



Italiadomani
PIANO NAZIONALE
DI RIPRESA E RESILIENZA



ecosustAinable effiCient tecH-driven Last miLE logiStics



- University of Naples Federico II
- Institute of System Analysis and Computer Science "Antonio Ruberti" of the National Research Council (IASI-CNR)
- University of Catania

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E-commerce

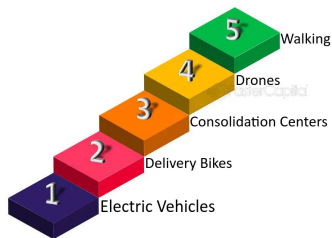
- The rapid growth of **e-commerce** has intensified the need for efficient last-mile delivery solutions



Environmental concerns

- Rise of **environmental concerns** and the emergence of **new business models** have prompted companies to explore alternative strategies to optimize last-mile delivery

Environmental Impact of Last-Mile Delivery



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Green Vehicles

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- One promising approach is the integration of **green vehicles** into the last-mile delivery process



Crowdsourcing

- Another approach is the integration of **crowdsourced drivers** into the last-mile delivery process



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Regular Vehicles and Occasional Drivers

- We develop a **model** to optimize last-mile delivery operations using a **combination of RVs and ODs**



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- 1 C. Archetti, M. Savelsbergh, M.G. Speranza, *The Vehicle Routing Problem with Occasional Drivers*, European Journal of Operational Research, 254, 2016, 472-480

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- ② C. Archetti, F. Guerriero, G. Macrina, *The online vehicle routing problem with occasional drivers*, Computers and Operations Research, 127, 2021
- ③ M. Boccia, A. Mancuso, A. Masone, C. Sterle, *Exact and heuristic approaches for the Truck-Drone Team Logistics Problem*, Transportation Research Part C: Emerging Technologies, 165, August 2024, 104691

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- ④ D. Yang, M.F. Hyland, R. Jayakrishnan, *Delivery Problem at Scale through a Set-Partitioning Formulation and Novel Decomposition Heuristic*, Transportation Research Part E: Logistics and Transportation Review, 188, August 2024, 103633

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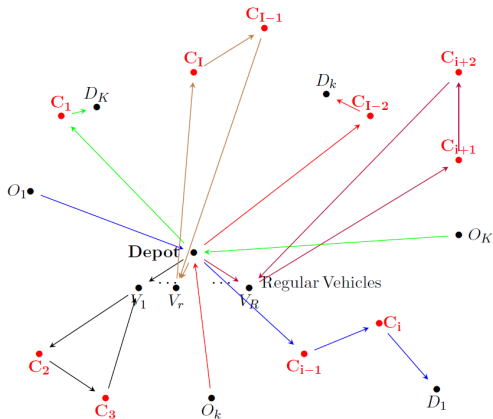


Figure: Network topology

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Assumptions:

- 1 All customers' requests have to be satisfied

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Assumptions:

- 1 All customers' requests have to be satisfied
- 2 Both ODs and RVs can serve more than one customer

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Assumptions:

- 1 All customers' requests have to be satisfied
- 2 Both ODs and RVs can serve more than one customer
- 3 Both ODs and RVs can use petrol/diesel vehicles or electric vehicles

The Supply Chain Network

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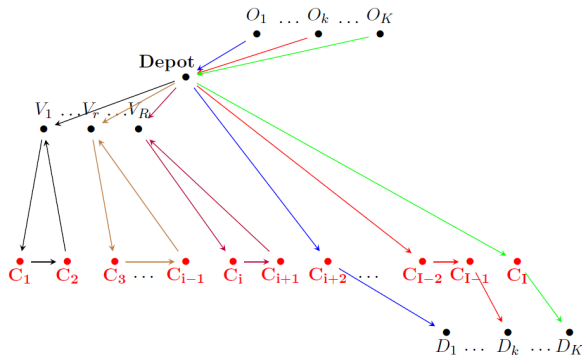


Figure: Supply chain representation

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Company's objective

$$\max \left\{ \sum_{i=1}^I \sum_{l=1}^2 p_i^l \left[\sum_{r=1}^R x_{ri}^l + \sum_{k=1}^K y_{ki}^l \right] + N \sum_{i=1}^I \left[\sum_{r=1}^R x_{ri}^2 + \sum_{k=1}^K y_{ki}^2 \right] \right\}$$

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Company's objective

$$\begin{aligned} \max \left\{ \sum_{i=1}^I \sum_{l=1}^2 p_i^l \left[\sum_{r=1}^R x_{ri}^l + \sum_{k=1}^K y_{ki}^l \right] + N \sum_{i=1}^I \left[\sum_{r=1}^R x_{ri}^2 + \sum_{k=1}^K y_{ki}^2 \right] \right. \\ \left. - \gamma(x, y) - \sum_{r=1}^R \sum_{l=1}^2 (c_r^{Bl} + c_r^{Ml} + c_r^{Sl}(x_r^l)) - \sum_{r=1}^R c_r^D \right. \\ \left. - \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^2 c_{ri}^l(x_{ri}^l) - \sum_{k=1}^K c^{OD} - \sum_{k=1}^K \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) \right\} \end{aligned}$$

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Company's objective

$$\begin{aligned} \max \left\{ \sum_{i=1}^I \sum_{l=1}^2 p_i^l \left[\sum_{r=1}^R x_{ri}^l + \sum_{k=1}^K y_{ki}^l \right] + N \sum_{i=1}^I \left[\sum_{r=1}^R x_{ri}^2 + \sum_{k=1}^K y_{ki}^2 \right] \right. \\ - \gamma(x, y) - \sum_{r=1}^R \sum_{l=1}^2 (c_r^{Bl} + c_r^{Ml} + c_r^{Sl}(x_r^l)) - \sum_{r=1}^R c_r^D \\ - \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^2 c_{ri}^l(x_{ri}^l) - \sum_{k=1}^K c^{OD} - \sum_{k=1}^K \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) \\ \left. - \sum_{i=1}^I \sum_{l=1}^2 P_i^l(\rho_i^l) \left[\sum_{r=1}^R d_{ri}^l + \sum_{k=1}^K d_{ki}^l \right] \right\} \end{aligned}$$

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Constraints

- Maximum **capacity** of each RV (per vehicle) of the company:

$$\sum_{i=1}^I x_{ri}^I \leq \bar{C}_r^I \quad \forall r = 1, \dots, R, \quad \forall I = 1, 2$$

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Constraints

- Maximum **capacity** of each RV (per vehicle) of the company:

$$\sum_{i=1}^I x_{ri}^l \leq \bar{C}_r^l \quad \forall r = 1, \dots, R, \quad \forall l = 1, 2$$

- Delay** (given by the difference between the sum of the delivery times to the customers preceding i and the time within which to make the delivery to i):

$$d_{ri}^l \geq \sum_{j=1}^I \delta_{ji}^l t_{rj}^l(x_r^l) - \bar{T}_i, \quad \forall r = 1, \dots, R, \quad \forall i = 1, \dots, I, \quad \forall l = 1, 2$$

$$d_{ri}^l \geq 0, \quad \forall r = 1, \dots, R, \quad \forall i = 1, \dots, I, \quad \forall l = 1, 2$$

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ODs' objective

$$\max \left\{ \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) - \sum_{l=1}^2 \tilde{c}_k^{Ml} - \sum_{l=1}^2 \tilde{c}_k^{Sl}(y_k^l) - \sum_{i=1}^I \sum_{l=1}^2 \tilde{c}_{ki}^l(y_{ki}^l) \right\}$$

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ODs' objective

$$\max \left\{ \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) - \sum_{l=1}^2 \tilde{c}_k^{Ml} - \sum_{l=1}^2 \tilde{c}_k^{Sl}(y_k^l) - \sum_{i=1}^I \sum_{l=1}^2 \tilde{c}_{ki}^l(y_{ki}^l) - \alpha_k \sum_{i=1}^I V_i d_{ki} \right\}$$

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Constraints

- Maximum **capacity** of OD k , at time t (for both types of vehicles):

$$\sum_{i=1}^I y_{ki}^I \leq \bar{C}_k^I \quad \forall I = 1, 2$$

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Constraints

- Maximum **capacity** of OD k , at time t (for both types of vehicles):

$$\sum_{i=1}^l y_{ki}^l \leq \bar{C}_k^l \quad \forall l = 1, 2$$

- Availability** of OD k to visit a customer with a vehicle of type l :

$$y_{ki}^l \leq M\beta_{ki}^l \quad \forall i = 1, \dots, l, \quad \forall l = 1, 2$$

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Constraints

- Maximum **capacity** of OD k , at time t (for both types of vehicles):

$$\sum_{i=1}^l y_{ki}^l \leq \overline{C}_k \quad \forall l = 1, 2$$

- Availability** of OD k to visit a customer with a vehicle of type l :

$$y_{ki}^l \leq M\beta_{ki}^l \quad \forall i = 1, \dots, I, \quad \forall l = 1, 2$$

- Delay** (given by the difference between the total delivery times and the time within which delivery to i must be made):

$$d_{ki}^l \geq \sum_{j=1}^I \delta_{ji}^{kl} t_{kj}^l(y_k^l) - \overline{T}_i, \quad \forall i = 1, \dots, I, \quad \forall l = 1, 2.$$

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Constraints

- Maximum **time available** for k , with vehicle l :

$$\sum_{i=1}^l t_{ki}^l(y_k^l) \leq \bar{T}_k^l, \quad \forall l = 1, 2.$$

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Customers' equilibrium conditions

$$p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^l) d_{ri}^l \begin{cases} = \rho_i^{l*} & \text{if } x_{ri}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } x_{ri}^{l*} = 0 \end{cases} \quad \forall r = 1, \dots, R, \quad \forall l = 1, 2$$

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$$p_i^l + \hat{c}_{ki}^l(y_{ki}^{l*}) - P_i^l(\rho_i^l) d_{ki}^l \begin{cases} = \rho_i^{l*} & \text{if } y_{ki}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } y_{ki}^{l*} = 0 \end{cases} \quad \forall k = 1, \dots, K, \quad \forall l = 1, 2$$

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$$p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^l) d_{ri}^l \begin{cases} = \rho_i^{l*} & \text{if } x_{ri}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } x_{ri}^{l*} = 0 \end{cases} \quad \forall r = 1, \dots, R, \quad \forall l = 1, 2$$

$$p_i^l + \hat{c}_{ki}^l(y_{ki}^{l*}) - P_i^l(\rho_i^l) d_{ki}^l \begin{cases} = \rho_i^{l*} & \text{if } y_{ki}^{l*} > 0 \\ \geq \rho_i^{l*} & \text{if } y_{ki}^{l*} = 0 \end{cases} \quad \forall k = 1, \dots, K, \quad \forall l = 1, 2$$

$$\phi_i^l(\rho_i^{l*}) \begin{cases} = \sum_{r=1}^R x_{ri}^{l*} + \sum_{k=1}^K y_{ki}^{l*} & \text{if } \rho_i^{l*} > 0 \\ \leq \sum_{r=1}^R x_{ri}^{l*} + \sum_{k=1}^K y_{ki}^{l*} & \text{if } \rho_i^{l*} = 0 \end{cases} \quad \forall l = 1, 2$$

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Definition (Variational Inequality Formulation)

Given a *continuous function* $F : K \rightarrow \mathbb{R}^N$, where $K \subset \mathbb{R}^N$ is a given (non empty) *convex* set, the *finite-dimensional variational inequality problem* consists in determining a vector $X^* \in K$ such that

$$\langle F(X^*), X - X^* \rangle = \sum_{i=1}^N F_i(X)(X_i - X_i^*) \geq 0, \quad \forall X \in K.$$

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Theorem

Let X^* be a *solution to the optimization problem*:

$$\min_K f(X)$$

where f is a continuously differentiable function and K is a closed convex set. Then, X^* is a *solution to the variational inequality problem* $VI(F, X)$:

$$\langle \nabla f(X^*), X - X^* \rangle \geq 0, \quad \forall X \in K,$$

where $F(X) \equiv \nabla f(X)$. Moreover, if $f(x)$ is a *convex* function and X^* is a solution to $VI(F, K)$, then X^* is a solution to the optimization problem.

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Theorem

If K is a *compact convex set*, then $VI(F, K)$ admits *at least one solution* X^* . Moreover, if $F(X)$ is *strictly monotone* on K , that is

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in K, X^1 \neq X^2,$$

the *solution is unique*.

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Define:

$$\begin{aligned} f^1(X^1) = & \sum_{i=1}^I \sum_{l=1}^2 p_i^l \left[\sum_{r=1}^R x_{ri}^l + \sum_{k=1}^K y_{ki}^l \right] + N \sum_{i=1}^I \left[\sum_{r=1}^R x_{ri}^2 + \sum_{k=1}^K y_{ki}^2 \right] \\ & - \gamma(x, y) - \sum_{r=1}^R \sum_{l=1}^2 (c_r^{Bl} + c_r^{Ml} + c_r^{Sl}(x_r^l)) - \sum_{r=1}^R c_r^D \\ & - \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^2 c_{ri}^l(x_{ri}^l) - \sum_{k=1}^K c^{OD} \\ & - \sum_{k=1}^K \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) - \sum_{i=1}^I P_i^l(\rho_i^l) \left[\sum_{r=1}^R d_{ri} + \sum_{k=1}^K d_{ki} \right], \end{aligned}$$

where $X^1 = (x, d^1) \in K^1 \subseteq \mathbb{R}^{I(R+1)}$

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Theorem

A vector $X^{1*} \in K^1$ is an *optimal solution* to the Delivery Company Optimization Problem if and only if is a solution to the Variational Inequality:

Find $X^{1*} \in K^1$ such that:

$$\langle \nabla f^1(X^{1*}), X^1 - X^{1*} \rangle \geq 0, \quad \forall X^1 \in K^1$$

Moreover, the variational inequality *admits at least one solution*.

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Theorem

Suppose that, for each costumer i , for each regular vehicle r , for each occasional driver k and for each mean of transport l , the cost functions c_r^{Sl} , c_{ri}^l , c_{ki}^l , P_i^l constitute families of **strictly monotone functions** w.r.t. their own variables. Moreover, suppose that, the depot management cost function γ is additive, that is:

$$\gamma(x, y) = \gamma^1(x) + \gamma^2(y)$$

and that γ is a strictly convex function. Then, the operator $F^1(X^1)$ is a strictly monotone function and the **solution** to the variational inequality is **unique**.

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Define:

$$\begin{aligned} f_k^2(X_k^2) = & \sum_{i=1}^I \sum_{l=1}^2 c_{ki}^l(y_{ki}^l) - \sum_{l=1}^2 \tilde{c}_k^{Ml} - \sum_{l=1}^2 \tilde{c}_k^{Sl}(y_k^l) \\ & - \sum_{i=1}^I \sum_{l=1}^2 \tilde{c}_{ki}^l(y_{ki}^l) - \alpha_k \sum_{i=1}^I V_i d_{ki} \end{aligned}$$

where $X_k^2 = (y_k, d_k^2) \in K_k^2 \subseteq \mathbb{R}^{3I}$

Theorem

A vector $X^{2*} = (X_k^{2*})_{k=1,\dots,K} \in K^2 = \prod_{k=1}^K K_k^2 \subseteq \mathbb{R}^{3KI}$ is an optimal solution to the optimization problems if and only if is a solution to the variational inequality:

Find $X^{2*} \in K^2$ such that:

$$\sum_{k=1}^K \sum_{i=1}^I \sum_{l=1}^2 \left[\frac{\partial \tilde{c}_{ki}^l(y_{ki}^{l*})}{\partial y_{ki}^l} + \frac{\partial \tilde{c}_k^{SI}(y_k^{l*})}{\partial y_{ki}^l} - \frac{\partial c_{ki}^l(y_{ki}^{l*})}{\partial y_{ki}^l} - p_i^l - \delta_l N \right] \times (y_{ki}^l - y_{ki}^{l*}) + \sum_{k=1}^K \sum_{i=1}^I [\alpha_k V_i] \times (d_{ki} - d_{ki}^*) \geq 0, \quad \forall X^2 \in K^2,$$

where $\delta_l = 1$ if $l = 2$, $\delta_l = 0$ otherwise. Moreover, such a variational inequality admits at least one solution.

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Define:

$$X^3 = (x, y, \rho) \in K^3 := \mathbb{R}^{I(RL+KL+1)}$$

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Theorem

A vector $X^{3*} \in K^3$ satisfies the *equilibrium conditions* if and only if it is a *solution to the variational inequality*:

Find $X^{3*} \in K^3$ such that:

$$\begin{aligned} & \sum_{r=1}^R \sum_{i=1}^I \sum_{l=1}^2 [p_i^l + \hat{c}_{ri}^l(x_{ri}^{l*}) - P_i^l(\rho_i^{l*})d_{ri}^{l*} - \rho_i^{l*}] \times (x_{ri}^l - x_{ri}^{l*}) \\ & + \sum_{k=1}^K \sum_{i=1}^I \sum_{l=1}^2 [p_i^l + \hat{c}_{ki}^l(y_{ki}^{l*}) - P_i^l(\rho_i^{l*})d_{ki}^{l*} - \rho_i^{l*}] \times (y_{ki}^l - y_{ki}^{l*}) \\ & + \sum_{i=1}^I \left[\sum_{r=1}^R x_{ri}^{l*} + \sum_{k=1}^K y_{ki}^{l*} - \phi_i^l(\rho_i^{l*}) \right] \times (\rho_i^l - \rho_i^{l*}) \geq 0, \quad \forall X^3 \in K^3 \end{aligned}$$

Moreover, such a variational inequality admits *at least one solution*.

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Definition (Network Equilibrium)

An *equilibrium state of the presented network* is reached if the flows between the tiers of the network coincide and the product shipments and prices satisfy the sum of the optimality conditions.

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Theorem

A vector $X^* = (X^{1*}, X^{2*}, X^{3*}) \in \mathcal{K}$ is an *equilibrium state of the network* if and only if it is a *solution to the VI given by the sum of the previous VIs*, where the feasible set is

$$\mathcal{K} := \left\{ X \in \mathbb{R}^{I(RL+R+KL+K+1)} : \text{all the constraints are satisfied} \right\}.$$

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Theorem

A vector $X^* = (X^{1*}, X^{2*}, X^{3*}) \in \mathcal{K}$ is an *equilibrium state of the network* if and only if it is a *solution to the VI given by the sum of the previous VIs*, where the feasible set is

$$\mathcal{K} := \left\{ X \in \mathbb{R}^{I(RL+R+KL+K+1)} : \text{all the constraints are satisfied} \right\}.$$

Theorem

Under the previous assumptions, the *solution* to the total VI is *unique*.

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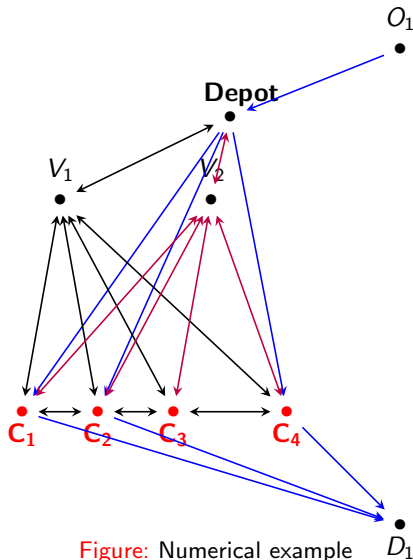
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Data

- 2 RDs with a petrol/diesel vehicle and an electric vehicle
- $\bar{C}_1^1 = 100$, $\bar{C}_1^2 = 70$, $\bar{C}_2^1 = 150$, $\bar{C}_2^2 = 90$
- 1 occasional driver with an electric vehicle
- $\bar{C}_k^2 = 60$
- 4 customers
- $\beta_{13}^1 = \beta_{13}^2 = 0$, $\beta_{11}^I = \beta_{12}^I = \beta_{14}^I = 1$

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- $\bar{C}_k^2 = 60$
- 4 customers
- $\beta_{13}^1 = \beta_{13}^2 = 0$, $\beta_{11}^I = \beta_{12}^I = \beta_{14}^I = 1$

Two Simulations

- S1:** without occasional drivers;
- S2:** with occasional drivers.

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S1	$i = 1$	$i = 2$	$i = 3$	$i = 4$	Objective Function
x_{1i}^{1*}	0	0	0	0	
x_{1i}^{2*}	18.84	8.69	24.52	17.95	
x_{2i}^{1*}	0	0	0	0	
x_{2i}^{2*}	21.16	11.31	15.48	12.05	
y_{1i}^{1*}	-	-	-	-	11,728
y_{1i}^{2*}	-	-	-	-	

Table: Optimal solutions in S1

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x_{2i}^{1*}	0	0	0	0	
x_{2i}^{2*}	21.16	11.31	15.48	12.05	
y_{1i}^{1*}	-	-	-	-	11,728
y_{1i}^{2*}	-	-	-	-	

Table: Optimal solutions in S1

S2	$i = 1$	$i = 2$	$i = 3$	$i = 4$	Objective Function
x_{1i}^{1*}	0	0	0	0	
x_{1i}^{2*}	5.53	2.66	26.44	11.70	
x_{2i}^{1*}	0	0	0	0	
x_{2i}^{2*}	5.33	2.55	13.56	5.87	
y_{1i}^{1*}	-	-	-	-	12,150
y_{1i}^{2*}	29.14	14.79	0	12.43	

Table: Optimal solutions in S2

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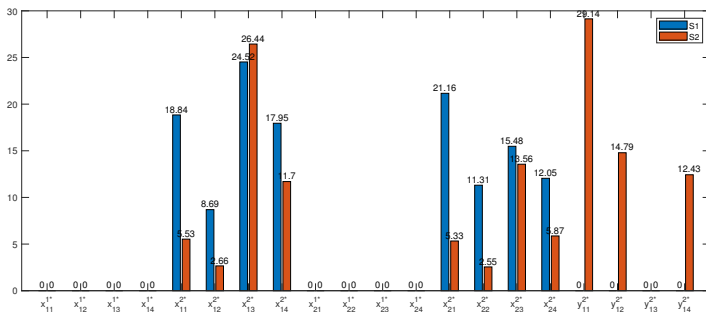


Figure: Comparison of optimal solutions in S1 and S2

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