International Human Migration Networks Under Regulations

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Joint work with **A. Nagurney**

Analysis, Control, and Numerics for PDE Models of Interest to Physical and Life Sciences

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Outline

1. Introduction

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Main Causes

Many social and economical factors affect the dynamics of human populations, such as poverty, violence, war, dictatorships, persecutions, oppression, genocide, ethnic cleansing, climate change, tsunamis, floods, earthquakes, famines, family reunification as well as economic and educational possibilities or a job.

Introduction

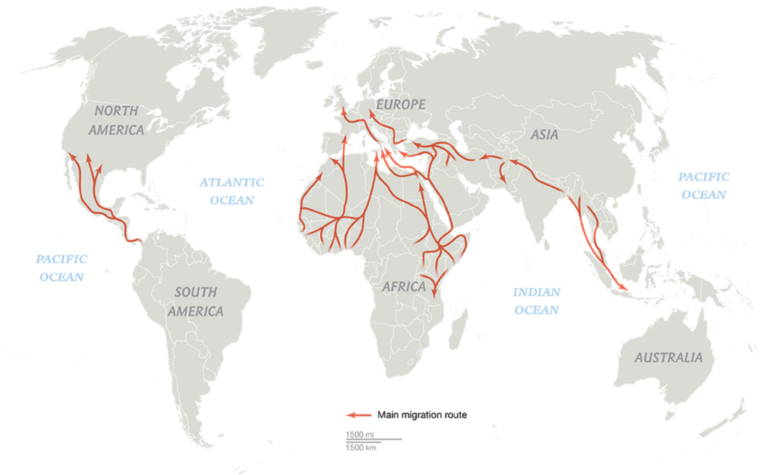

Figure: World's congested human migration routes

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From 2018 until January 2019, 17% of arrivals by sea were registered in Italy, compared to 69% in 2017 (UNHCR).

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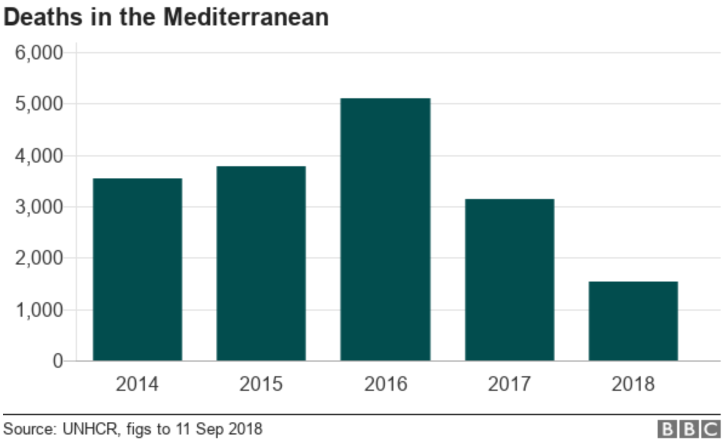

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Now, with the **COVID-19 pandemic**, declared by the World Health Organization on March 11, 2020, Dolmans et al. (2020) report that COVID-19 is likely to exacerbate what is already a humanitarian emergency in terms of a global refugee crisis. The authors argue that refugees may encounter increased difficulty in seeking asylum due to measures imposed by governments in response to the pandemic.

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- Cui and Bai, 2014: evolution of population density and spread of epidemics in population systems.
- Volpert, Petrovskiic, Zincenkoc, 2017: interaction of human migration and wealth distribution.

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- Nagurney, Daniele 2021: development of a network model with regulations.

The mathematical model

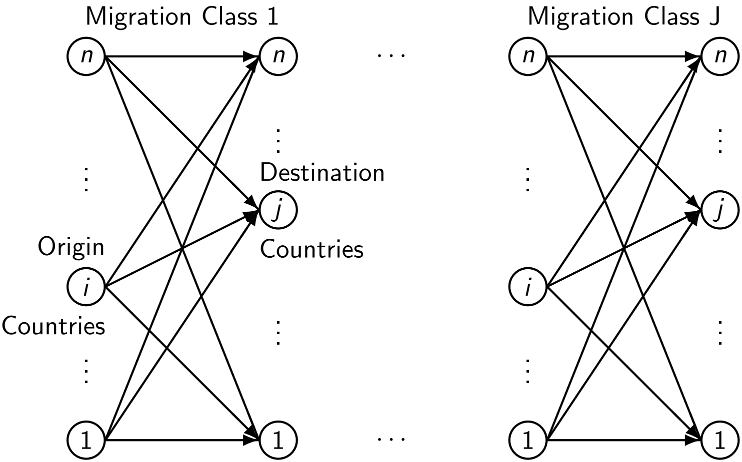

Figure: The Network Structure of International Human Migration

Common Notation

Notation	Definition
f_{ij}^k	the flow of migrants of class k from country i to country j. The $\{f_{ij}^k\}$ elements for all i and j and fixed k are grouped into the vector $f^k \in R_+^{nn}$. We then further group the f^k vectors; $k = 1, \ldots, J$, into the vector $f \in R_+^{Jnn}$.
p_i^k	the nonnegative population of migrant class k in country i. We group the populations of class k; $k = 1, \ldots, J$, into the vector $p^k \in R_+^n$. We then further group all such vectors into the vector $p \in R_+^{Jn}$.
$\bar{p}_i^k$	the initial fixed population of class k in country i; $i = 1, \ldots, n$; $k = 1, \ldots, J$.
$u_i^k(p)$	the utility perceived by class k in country i; $i = 1, \ldots, n$; $k = 1, \ldots, J$.
$c_{ij}^k(f)$	the cost of international migration, which includes economic, psychological, and social costs encumbered by class k in migrating from country i to country j; $i = 1, \ldots, n$; $j = 1, \ldots, n$; $k = 1, \ldots, J$.

The mathematical model

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Conservation of flow equations

$$\bar{p}_i^k = \sum_l f_{il}^k, \ (a) \quad \text{and} \quad p_i^k = \sum_l f_{li}^k, \ (b) \quad \forall i, \forall k$$

$$\Updownarrow$$

$$p_i^k - \bar{p}_i^k = \sum_l f_{li}^k - \sum_l f_{il}^k, \quad \forall i, \forall k$$

Equilibrium Conditions

Definition (International Human Migration Equilibrium without Regulations)

A vector of populations and international migration flows $(p^*, f^*) \in K^1$ is in equilibrium if it satisfies the equilibrium conditions: For each class k; $k = 1, \ldots, J$ and each pair of countries i, j; $i = 1, \ldots, n$; $j = 1, \ldots, n$:

$$u_i^k(p^*) + c_{ij}^k(f^*) \begin{cases} = u_j^k(p^*) - \lambda_i^{k*}, & \text{if} \quad f_{ij}^{k*} > 0 \\ \geq u_j^k(p^*) - \lambda_i^{k*}, & \text{if} \quad f_{ij}^{k*} = 0 \end{cases}$$

and

$$\lambda_i^{k*} \begin{cases} \geq 0, & \text{if} \quad \sum_{l \neq i} f_{il}^{k*} = \bar{p}_i^k, \\ = 0, & \text{if} \quad \sum_{l \neq i} f_{il}^{k*} < \bar{p}_i^k \end{cases}$$

Variational Formulation of the International Human Migration Model without Regulations

Theorem

A population and migration flow pattern $(p^*, f^*) \in K^1$ is an international human migration equilibrium without regulations according to Definition 1, if and only if it satisfies the variational inequality problem

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0,$$

$$\forall (p, f) \in K^1 \equiv \{(p, f) | f \geq 0, \ \textit{and} \, (a) \ \textit{and} \, (b) \ \textit{hold}\}$$

We now consider regulations imposed by a single country $\bar{j}$:

$$\sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^k \leq U_{\bar{j}} \qquad (c)$$

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Different types of regulations

- restriction of the migratory flow from a specific country $\bar{i}$ and specific class of migrant $\bar{k}$: $f_{\bar{i}\bar{j}}^{\bar{k}} \leq U_{\bar{j}}$

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Different types of regulations

- restriction of the migratory flow from a specific country $\bar{i}$ and specific class of migrant $\bar{k}$: $f_{\bar{i}\bar{j}}^{\bar{k}} \leq U_{\bar{j}}$
- upper bounds on all incoming migrants from a specific country $\bar{i}$, irrespective of class: $\sum_k f_{\bar{i}\bar{j}}^k \leq U_{\bar{j}}$

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- restriction of the migratory flow from a specific country $\bar{i}$ and specific class of migrant $\bar{k}$: $f_{\bar{i}\bar{j}}^{\bar{k}} \leq U_{\bar{j}}$
- upper bounds on all incoming migrants from a specific country $\bar{i}$, irrespective of class: $\sum_k f_{\bar{i}\bar{j}}^k \leq U_{\bar{j}}$
- regulations restricting the number of all incoming migrants of class $\bar{k}$ from a group of countries: $\sum_{i \in C^1} f_{i\bar{j}}^{\bar{k}} \leq U_{\bar{j}}$

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New Feasible Set

$$K^2 \equiv K^1 \cap \{f | (c) \text{ is satisfied}\}$$

Variational Formulation of the International Human Migration Model with Regulations

New Feasible Set

$$K^2 \equiv K^1 \cap \{f | (c) \text{ is satisfied}\}$$

Theorem

A population and migration flow pattern $(p^*, f^*) \in K^2$ is an international human migration equilibrium with regulations, if and only if it satisfies the variational inequality problem

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0, \quad \forall (p, f) \in K^2$$

Equivalent Variational Inequality Formulation

VI in flows

Determine $f^* \in K^3 \equiv \{f | f \in R_+^{Jnn}$ and (a) and (c) hold$\}$ such that

$$\sum_i \sum_j \sum_k (-\hat{u}_j^k(f^*) + c_{ij}^k(f^*)) \times (f_{ij}^k - f_{ij}^{k*}) \geq 0, \quad \forall f \in K^3$$

Lagrange Function

K^3 can be rewritten as follows:

$$K^3 = \left\{ f : -f \leq 0; \sum_j f_{ij}^k - \bar{p}_i^k = 0, \ \forall i, \ \forall k; \sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^k - U_{\bar{j}} \leq 0 \right\}$$

and the last variational inequality can be rewritten as a minimization problem, since if we set:

$$V(f) = \sum_i \sum_j \sum_k (-\hat{u}_j^k(f^*) + c_{ij}^k(f^*)) \times (f_{ij}^k - f_{ij}^{k*}),$$

then we have:

$$V(f) \geq 0 \text{ for } f \in K^3 \text{ and } \min_{f \in K^3} V(f) = V(f^*) = 0.$$

Existence of Lagrange Multipliers and Strong Duality

Theorem

If $f^* \in K^3$ is a solution to variational inequality, then the Lagrange multipliers $\bar{\gamma} \in R_+^{Jnn}$, $\bar{\delta} \in R^{Jn}$, and $\bar{\mu}_{\bar{j}} \in R_+$ do exist, and for all i, j, k, and $\bar{j}$, the following conditions hold true:

$$\bar{\gamma}_{ij}^k(-f_{ij}^{k*}) = 0, \quad \bar{\delta}_{ik}\left(\sum_j f_{ij}^{k*} - \bar{p}_i^k\right) = 0,$$
$$\bar{\mu}_{\bar{j}}\left(\sum_{i \in C^1}\sum_{k \in C^1} f_{i\bar{j}}^{k*} - U_{\bar{j}}\right) = 0 \qquad (d)$$

$$-\hat{u}_j^k(f^*) + c_{ij}^k(f^*) - \bar{\gamma}_{ij}^k + \bar{\delta}_{ik} = 0, \quad \text{if } j \neq \bar{j} \qquad (e)$$

$$-\hat{u}_{\bar{j}}^k(f^*) + c_{i\bar{j}}^k(f^*) - \bar{\gamma}_{i\bar{j}}^k + \bar{\delta}_{ik} + \bar{\mu}_{\bar{j}} = 0, \quad \text{if } j = \bar{j} \qquad (f)$$

Moreover, the strong duality also holds true; namely:

$$V(f^*) = \min_{f \in K^3} V(f) = \max_{\gamma \in R_+^{Jnn}, \delta \in R^{Jn}, \mu_{\bar{j}} \in R_+} \min_{f \in R^{Jnn}} \mathcal{L}(f, \gamma, \delta, \mu_{\bar{j}})$$

Interpretation of the Lagrange Analysis

- If $f_{ij}^{k*} > 0$ for some $j \neq \bar{j}$; from (d) we know that then $\bar{\gamma}_{ij}^k = 0$. It then follows from (e) that: $\bar{\delta}_{ik} + c_{ij}^k(f^*) = \hat{u}_j^k(f^*)$

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- If $f_{ij}^{k*} = 0$, for some $j \neq \bar{j}$, then $\bar{\gamma}_{ij}^k \geq 0$, and, from (e), we can infer that:

$$\bar{\delta}_{ik} + c_{ij}^k(f^*) = \hat{u}_j^k(f^*) + \bar{\gamma}_{ij}^k \quad \text{equivalently:} \quad \bar{\delta}_{ik} + c_{ij}^k(f^*) \geq \hat{u}_j^k(f^*)$$

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$$\bar{\delta}_{ik} + c_{ij}^k(f^*) = \hat{u}_j^k(f^*) + \bar{\gamma}_{ij}^k \quad \text{equivalently:} \quad \bar{\delta}_{ik} + c_{ij}^k(f^*) \geq \hat{u}_j^k(f^*)$$

- If $f_{ii}^* > 0$, then from (d) it follows, since $c_{ii} = 0$, that

$$0 + \bar{\delta}_{ik} = \hat{u}_i^k(f^*) \quad \text{and, hence,} \quad \bar{\delta}_{ik} = \hat{u}_i^k(f^*).$$

Interpretation of the Lagrange Analysis

If we consider a destination node $\bar{j}$, under the regulation, and, if $f_{i\bar{j}}^{k*} > 0$, then (d) applies and we obtain:

$$\bar{\delta}_{ik} + c_{i\bar{j}}^{k}(f^*) = \hat{u}_{\bar{j}}^{k}(f^*) - \bar{\mu}_{\bar{j}}$$

If the upper bound holds tightly, then the migrants incur a higher utility at destination node $\bar{j}$ than just the sum of the origin node utility and the migration cost.

Alternative Variational Inequality Formulation

Variational Inequality

determine $(f^*, \delta^*, \mu_{\bar{j}}^*) \in K^4$ such that

$$\sum_{i; i \notin C^1} \sum_{j \neq \bar{j}} \sum_{k; k \notin C^1} \left(-\hat{u}_j^k(f^*) + c_{ij}^k(f^*) + \delta_{ik}^*\right) \times \left(f_{ij}^k - f_{ij}^{k*}\right)$$

$$+ \sum_{i \in C^1} \sum_{k \in C^1} \left(-\hat{u}_{\bar{j}}^k(f^*) + c_{i\bar{j}}^k(f^*) + \delta_{ik}^* + \mu_{\bar{j}}^*\right) \times \left(f_{i\bar{j}}^k - f_{i\bar{j}}^{k*}\right)$$

$$+ \sum_i \sum_k \left(\bar{p}_i^k - \sum_j f_{ij}^{k*}\right) \times \left(\delta_{ik} - \delta_{ik}^*\right)$$

$$+ \left(U_{\bar{j}} - \sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^{k*}\right) \times \left(\mu_{\bar{j}} - \mu_{\bar{j}}^*\right) \geq 0,$$

$$\forall (f, \delta, \mu_{\bar{j}}) \in K^4 \equiv \{(f, \delta, \mu_{\bar{j}}) | f \in R_+^{Jnn}, \delta \in R^{Jn}, \mu_{\bar{j}} \in R_+\}$$

Standard Form

Positions

- $\mathcal{K} \equiv K^4$
- $X \equiv (f, \delta, \mu_{\bar{j}})$
- $N = Jnn + Jn + 1$
- $F \equiv (F_1, F_2, F_3, F_4)$: the components of F_1 consist of the elements: $-\hat{u}_j^k(f) + c_{ij}^k(f) + \delta_{ik}$, for $i; i \notin C^1$ and $j \neq \bar{j}$, and $k; k \notin C^1$; the components of F_2 consist of the elements: $-\hat{u}_{\bar{j}}^k(f) + c_{i\bar{j}}^k(f) + \delta_{ik} + \mu_{\bar{j}}$, for $i \in C^1$ and $k \in C^1$; F_3 consists of the elements: $\bar{p}_i^k - \sum_j f_{ij}^k$, $\forall i, k$, and, finally, F_4 consists of the single element: $U_{\bar{j}} - \sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^k$

Standard Variational Inequality

Determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}$$

Illustrative Examples

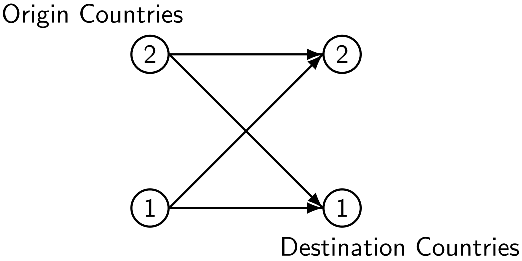

Figure: International Migration Network for Illustrative Examples

Case without Regulations

Data

- $\bar{p}_1 = 50$ and $\bar{p}_2 = 0$
- $u_1(p) = -p_1 + 100$ and $u_2(p) = -p_2 + 120$
- $c_{11}(f) = c_{22}(f) = 0$, $c_{12}(f) = .1f_{12} + 7$, $c_{21}(f) = f_{21} + 10$

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- $c_{11}(f) = c_{22}(f) = 0,\ c_{12}(f) = .1f_{12} + 7,\ c_{21}(f) = f_{21} + 10$

Equilibrium Solution

- $f^*_{12} = 30, \quad f^*_{11} = 20, \quad f^*_{21} = 0, \quad f^*_{22} = 0$
- $p^*_1 = 20, \quad p^*_2 = 30$
- $\hat{u}_1(f^*) = u_1(p^*) = 80, \quad \hat{u}_2(f^*) = u_2(p^*) = 90$
- $c_{11}(f^*) = c_{22}(f^*) = 0,\ c_{12}(f^*) = 10$, and $c_{21}(f^*) = 10$
- $\bar{\delta}_{11} = 80,\ \bar{\delta}_{21} = 90$, and $\bar{\gamma}_{11} = \bar{\gamma}_{21} = \bar{\gamma}_{12} = \bar{\gamma}_{22} = 0$

and the equilibrium conditions hold true.

Case with regulations

Regulation

$$f_{1\bar{2}} \leq U_{\bar{2}} = 20$$

Case with regulations

Regulation

$$f_{1\bar{2}} \leq U_{\bar{2}} = 20$$

New Equilibrium Solution

- $f^*_{11} = 30, \quad f^*_{1\bar{2}} = 20, \quad f^*_{21} = 0, \quad f^*_{2\bar{2}} = 0$
- $p^*_1 = 30$ and $p^*_{\bar{2}} = 20$
- $\hat{u}_1(f^*) = u_1(p^*) = 70, \quad \hat{u}_{\bar{2}}(f^*) = u_{\bar{2}}(p^*) = 100$
- $c_{11}(f^*) = c_{2\bar{2}}(f^*) = 0;\ c_{1\bar{2}}(f^*) = 9,\ c_{21}(f^*) = 10$
- $\bar{\mu}_{\bar{2}} = 21$
- $\bar{\delta}_{11} = 70,\ \bar{\delta}_{21} = 100$, and $\bar{\gamma}_{11} = \bar{\gamma}_{21} = \bar{\gamma}_{12} = \bar{\gamma}_{22} = 0$

and the equilibrium conditions hold true.

The Modified Projection Method

- **Step 0: Initialization** Initialize with $X^0 \in \mathcal{K}$. Set $t := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{L}$, where L is the Lipschitz constant
- **Step 1: Computation** Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \bar{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{K}$$

- **Step 2: Adaptation** Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + \beta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}$$

- **Step 3: Convergence Verification**
  If $|X^t - X^{t-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $t := t + 1$ and go to Step 1

Explicit Formulae

$$\bar{f}_{ij}^{kt} = \max\{0, f_{ij}^{k(t-1)} + \beta(\hat{u}_j^k(f^{t-1}) - c_{ij}^k(f^{t-1}) - \delta_{ik}^{t-1})\}, \ i \notin C^1; j \neq \bar{j}; k \notin C^1$$

$$\bar{f}_{i\bar{j}}^{kt} = \max\{0, f_{i\bar{j}}^{k(t-1)} + \beta(\hat{u}_{\bar{j}}^k(f^{t-1}) - c_{i\bar{j}}^k(f^{t-1}) - \delta_{ik}^{t-1} - \mu_{\bar{j}}^{t-1})\}, \ i \in C^1; k \in C$$

$$\bar{\delta}_{ik}^t = \delta_{ik}^{t-1} + \beta\Big(\sum_j f_{ij}^{k(t-1)} - \bar{p}_i^k\Big), \ \forall i, \forall k$$

$$\bar{\mu}_{\bar{j}}^t = \max\{0, \mu_{\bar{j}}^{t-1} + \beta\Big(\sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^{k(t-1)} - U_{\bar{j}}\Big)\}$$

Single Class Example without and with a Regulation

Example 1

3 countries, no regulations and a single class of migrants

Single Class Example without and with a Regulation

Example 1

3 countries, no regulations and a single class of migrants

Data

- $\bar{p}_1 = 10,000$, $\bar{p}_2 = 5,000$, and $\bar{p}_3 = 1,000$
- $u_1(p) = -p_1 - .5p_2 + 30,000$, $u_2(p) = -2p_2 - p_1 + 20,000$, $u_3(p) = -3p_3 + .5p_2 + 10,000$
- $c_{ii} = 0, \quad i = 1, 2, 3$
- $c_{12}(f) = 2f_{12} + 20, \quad c_{13}(f) = f_{13} + 30$
- $c_{21}(f) = 5f_{21} + 40, \quad c_{23}(f) = 4f_{23} + 20$
- $c_{31}(f) = 6f_{31} + 80, \quad c_{32}(f) = 4f_{32} + 60$
- $\beta = .1$

Single Class Example without Regulations

Equilibrium migration flow pattern

- $f^*_{11} = 10,000.00, \quad f^*_{12} = 0.00, \quad f^*_{13} = 0.00$
  $f^*_{21} = 2,447.90, \quad f^*_{22} = 1,519.66, \quad f^*_{23} = 1,032.44$
  $f^*_{31} = 1,000.00, \quad f^*_{32} = 0.00, \quad f^*_{33} = 0.00$
- $c_{11}(f^*) = 0.00, \quad c_{12}(f^*) = 20.00, \quad c_{13}(f^*) = 30.00$
  $c_{21}(f^*) = 12,279.50, \quad c_{22}(f^*) = 0.00, \quad c_{23}(f^*) = 4,149.76$
  $c_{31}(f^*) = 6,080.09, \quad c_{32}(f^*) = 60.00, \quad c_{33}(f^*) = 0.00$
- $p^*_1 = 13,447.92, \quad p^*_2 = 1,519.66, \quad p^*_3 = 1,032.44$
- $u_1(p^*) = 15,792.25, \quad u_2(p^*) = 3,512.75, \quad u_3(p^*) = 7,662.51$
- $\delta^*_1 = 15,792.25, \quad \delta^*_2 = 3,512.75, \quad \delta^*_3 = 9,712.17$

Single Class Example with a Regulation

Regulation

$$f_{2\bar{1}} + f_{3\bar{1}} \leq 2,000$$

Equilibrium migration pattern

- $f^*_{11} = 10,000.00, \quad f^*_{12} = 0.00, \quad f^*_{13} = 0.00,$
  $f^*_{2\bar{1}} = 1,458.43, \quad f^*_{22} = 2,545.92, \quad f^*_{23} = 995.65$
  $f^*_{3\bar{1}} = 541.59, \quad f^*_{32} = 0.00, \quad f^*_{33} = 458.41$
- $c_{11}(f^*) = 0.00, \quad c_{12}(f^*) = 20.00, \quad c_{13}(f^*) = 30.00$
  $c_{2\bar{1}}(f^*) = 7,332.13, \quad c_{22}(f^*) = 0.00, \quad c_{23}(f^*) = 4,002.61$
  $c_{3\bar{1}}(f^*) = 3,329.52, \quad c_{32}(f^*) = 60.00, \quad c_{33}(f^*) = 0.00$
- $p^*_1 = 12,000.02, \quad p^*_2 = 2,545.92, \quad p^*_3 = 1,454.06$
- $u_{\bar{1}}(p^*) = 16,727.02, \quad u_2(p^*) = 2,908.15, \quad u_3(p^*) = 6,910.76$
- $\delta^*_1 = 16,727.02, \quad \delta^*_2 = 2,908.15, \quad \delta^*_3 = 6,910.76$
- $\mu^*_{\bar{1}} = 6,486.74$

Multiclass Example without Regulations

Example 2

3 countries and 2 classes of migrants

Multiclass Example without Regulations

Example 2

3 countries and 2 classes of migrants

Data

- $u_1^1(p) = -p_1^1 - .5p_2^1 - .5p_1^2 + 30,000$, $u_2^1(p) = -2p_2^1 - p_1^1 - p_2^2 + 20,000$, $u_3^1(p) = -3p_3^1 + .5p_2^1 - p_3^2 + 10,000$ $u_1^2(p) = -2p_1^2 - p_1^1 + 25,000$, $u_2^2(p) = -3p_2^2 - p_2^1 + 15,000$, $u_3^2(p) = -p_3 - .5p_1^1 + 20,000$
- $\bar{p}_1^2 = 5,000$, $\bar{p}_2^2 = 3,000$, and $\bar{p}_3^2 = 500$
- $c_{ii}^k = 0$, $\forall i$, and for $k = 1, 2$
- $c_{12}^2(f) = 2f_{12}^2 + 10$, $c_{13}^2(f) = f_{13}^2 + 20$, $c_{21}^2(f) = 3f_{21}^2 + 10$, $c_{23}^2(f) = 2f_{23}^2 + 30$, $c_{31}^2(f) = f_{31}^2 + 25$, $c_{32}^2(f) = 2f_{32}^2 + 15$

Multiclass Example without Regulations

Equilibrium migration pattern

- $f_{11}^{1*} = 10,000.00, \quad f_{12}^{1*} = 0.00, \quad f_{13}^{1*} = 0.00,$
  $f_{21}^{1*} = 2,649.57, \quad f_{22}^{1*} = 1,547.75, \quad f_{23}^{1*} = 802.68,$
  $f_{31}^{1*} = 1,000.00, \quad f_{32}^{1*} = 0.00, \quad f_{33}^{1*} = 0.00$
- $f_{11}^{2*} = 2,343.67, \quad f_{12}^{2*} = 182.49, \quad f_{13}^{2*} = 2,473.85,$
  $f_{21}^{2*} = 0.00, \quad f_{22}^{2*} = 1,955.57, \quad f_{23}^{2*} = 1,044.43,$
  $f_{31}^{2*} = 0.00, \quad f_{32}^{2*} = 0.00, \quad f_{33}^{2*} = 500.00$
- $c_{11}^1(f^*) = 0.00, \quad c_{12}^1(f^*) = 20.00, \quad c_{13}^1(f^*) = 30.00,$
  $c_{21}^1(f^*) = 13,287.86, \quad c_{22}^1(f^*) = 0.00, \quad c_{23}^1(f^*) = 3,230.72,$
  $c_{31}^1(f^*) = 6,080.09, \quad c_{32}^1(f^*) = 60.00, \quad c_{33}^1(f^*) = 0.00$

Multiclass Example without Regulations

Equilibrium migration pattern

- $c^2_{11}(f^*) = 0.00, \quad c^2_{12}(f^*) = 374.98, \quad c^2_{13}(f^*) = 2,493.85,$
  $c^2_{21}(f^*) = 10.00, \quad c^2_{22}(f^*) = 0.00, \quad c^2_{23}(f^*) = 2,118.86,$
  $c^2_{31}(f^*) = 25.00, \quad c^2_{32}(f^*) = 15.00, \quad c^2_{33}(f^*) = 0.00$
- $p^{1*}_1 = 13,649.59, \quad p^{1*}_2 = 1,547.75, \quad p^{1*}_3 = 802.68,$
  $p^{2*}_1 = 2,343.67, \quad p^{2*}_2 = 2,138.06, \quad p^{2*}_3 = 4,018.28$
- $u^1_1(p^*) = 14,404.70, \quad u^1_2(p^*) = 1,116.84, \quad u^1_3(p^*) = 4,347.56,$
  $u^2_1(p^*) = 6,663.08, \quad u^2_2(p^*) = 7,038.06, \quad u^2_3(p^*) = 9,156.92$
- $\delta^{1*}_1 = 14,404.70, \quad \delta^{1*}_2 = 1,116.84, \quad \delta^{1*}_3 = 8,324.62,$
  $\delta^{2*}_1 = 6,663.08, \quad \delta^{2*}_2 = 7,038.06, \quad \delta^{2*}_3 = 9,156.92$

Muliticlass Example with a Regulation

Regulation

$$f^1_{13} + f^1_{23} + f^2_{13} + f^2_{23} \leq 2,000$$

Equilibrium migration pattern

- $f^{1*}_{11} = 10,000.00, \quad f^{1*}_{12} = 0.00, \quad f^{1*}_{13} = 0.00,$
  $f^{1*}_{21} = 2,746.92, \quad f^{1*}_{22} = 1,788.86, \quad f^{1*}_{23} = 464.22,$
  $f^{1*}_{31} = 1,000.00, \quad f^{1*}_{32} = 0.00, \quad f^{1*}_{33} = 0.00$
- $f^{2*}_{11} = 3,581.93, \quad f^{2*}_{12} = 232.53, \quad f^{2*}_{13} = 1,185.54,$
  $f^{2*}_{21} = 0.00, \quad f^{2*}_{22} = 2,649.76, \quad f^{2*}_{23} = 350.24,$
  $f^{2*}_{31} = 0.00, \quad f^{2*}_{32} = 0.00, \quad f^{2*}_{33} = 500.00$
- $c^1_{11}(f^*) = 0.00, \quad c^1_{12}(f^*) = 20.00, \quad c^1_{13}(f^*) = 30.00,$
  $c^1_{21}(f^*) = 13,774.62, \quad c^1_{22}(f^*) = 0.00, \quad c^1_{23}(f^*) = 1,876.90,$
  $c^1_{31}(f^*) = 6.080.02, \quad c^1_{32}(f^*) = 60.00, \quad c^1_{33}(f^*) = 0.00$

Muliticlass Example with a Regulation

Equilibrium migration pattern

- $c^2_{11}(f^*) = 0.00, \quad c^2_{12}(f^*) = 475.06, \quad c^2_{13}(f^*) = 1,205.54,$
  $c^2_{21}(f^*) = 10.00, \quad c^2_{22}(f^*) = 0.00, \quad c^2_{23}(f^*) = 730.47,$
  $c^2_{31}(f^*) = 25.00, \quad c^2_{32}(f^*) = 15.00, \quad c^2_{33}(f^*) = 0.00$
- $p^{1*}_1 = 13,746.93, \quad p^{1*}_2 = 1,788.86, \quad p^{1*}_3 = 464.22,$
  $p^{2*}_1 = 3,581.93, \quad p^{2*}_2 = 2,882.29, \quad p^{2*}_3 = 2,035.78$
- $u^1_1(p^*) = 13,567.68, \quad u^1_2(p^*) = -206.93, \quad u^1_3(p^*) = 7,465.98,$
  $u^2_1(p^*) = 4,089.21, \quad u^2_2(p^*) = 4,564.27, \quad u^2_3(p^*) = 11,090.76$
- $\delta^{1*}_1 = 13,567.69, \quad \delta^{1*}_2 = -206.94, \quad \delta^{1*}_3 = 7,487.66,$
  $\delta^{2*}_1 = 4,089.20, \quad \delta^{2*}_2 = 4,564.27, \quad \delta^{2*}_3 = 11,090.75,$
- $\mu^*_3 = 5,796.02$

Conclusions

- Challenges such as climate change and associated disruptions, along with wars, conflicts, and strife, are acting as push forces for humans to seek locations of greater safety and security

Conclusions

- Challenges such as climate change and associated disruptions, along with wars, conflicts, and strife, are acting as push forces for humans to seek locations of greater safety and security
- Governments are being forced to deal with increases in migratory flows across national boundaries