An optimization model for the management of green areas

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Joint work with **D. Sciacca**

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Outline

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- **Intergovernmental Panel of Climate Change**: most of the observed increase in globally averaged temperatures since the mid-twentieth century is very likely due to the observed increase in anthropogenic greenhouse gas concentrations
- **Consequences**: global warming, climate change and a progressive environmental degradation (heat stress, storms and extreme precipitation, air pollution, melting glaciers, sea level rise, extinctions, reduced water resources, etc.)

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International Agreements

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- To enhance infrastructure and transport, thereby promoting the movement through public transport and the sharing of mobility (car sharing) and reducing the consumption of private cars
- To promote the purchase of zero-emission vehicles or to allow the movement of vehicles with alternate plates
- To promote new technologies for monitoring air pollution such as air quality monitoring networks

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Good Solutions

- To increase public space that can be used to create new parks and open spaces for both recreational and commercial purposes

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Costs

- To acquire new spaces
- To train personnel
- To convert land in green areas
- To manage pre-existent types of green areas

The Mathematical Model

- n cities
- m different types of green areas
- $x_{ij} \in \mathbb{R}_+$: space of green area of type j in city i
- $\underline{x}_{ij} \in \mathbb{R}_+$: the pre-existent space of green area of type j in city i

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We assume that the optimal space of green area is not less than a quantity, l_i, imposed by law and, if in a country such a constraint does not exist, then the minimum quantity of green area imposed by law corresponds to the pre-existing one.

$$m_i := \max\left\{ l_i, \sum_{j=1}^{m} \underline{x}_{ij} \right\}, \quad \forall i = 1, \ldots, n$$

The Mathematical Model

Condition 1

$$\sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} \underline{x}_{ij} \geq m_i, \quad \forall i = 1, \ldots, n$$

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$$\sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} \underline{x}_{ij} \geq m_i, \quad \forall i = 1, \ldots, n$$

We also impose that the total surface of green area in every city i is less than a fixed quantity u_i related to the total area of the city:

Condition 2

$$\sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} \underline{x}_{ij} \leq u_i, \quad \forall i = 1, \ldots, n$$

The Mathematical Model

Variables and Functions

- $f_i \in \mathbb{R}_+$: flow of urban circulation in city i
- $c^a_{ij} = c^a_{ij}(x_{ij}, f_i)$: acquisition cost associated with the expansion of green area of type j in city i
- g_i : quantity of employees to train for the management and maintenance of public green spaces
- $c_i^T = c_i^T(g_i)$: training cost for such personnel
- $c^t_{ij} = c^t_{ij}(x_{ij})$: transformation cost which is required to convert the land in green area of type j in city i
- $\gamma^m_{ij} = \gamma^m_{ij}(x_{ij}, g_i)$: management cost of green area of type j in city i
- $\underline{\gamma}^m_{ij}$: management cost of pre-existent types of green areas

The Mathematical Model

Total Management Costs

$$c_{ij}^m = \underline{\gamma}_{ij}^m + \gamma_{ij}^m(x_{ij}, g_i), \quad \forall i = 1, \dots, n, \ \forall j = 1, \dots, m$$

The Mathematical Model

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Additional Functions

- $e_i(f_i) = \sum_{k=1}^{2} e_{ik} + e_{i3}(f_i)$: total amount of CO_2 emissions of city i
- $e_i^a = \sum_{j=1}^{m} \alpha_j x_{ij}$: quantity of CO_2 absorbed by the overall green area in city i

The Mathematical Model

Purpose

Minimize the total costs incurred by the external institution to adapt the green area surface in each city to its real needs

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Optimality Problem

$$\min \left\{ \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}^{a}(x_{ij}, f_i) + \sum_{i=1}^{n} c_i^{T}(g_i) + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}^{t}(x_{ij}) + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}^{m}(x_{ij}, g_i) \right\}$$

The Mathematical Model

Constraints

- $x_{ij} \geq \underline{x}_{ij}, \quad \forall i, \ \forall j$
- $g_i, \ f_i \geq 0, \quad \forall i$
- $m_i \leq \sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} \underline{x}_{ij} \leq u_i, \quad \forall i$
- $\sum_{j=1}^{m} \alpha_j x_{ij} \geq \sum_{k=1}^{2} e_{ik} + e_{i3}(f_i) - \sum_{j=1}^{m} \alpha_j \underline{x}_{ij}, \quad \forall i$
- $g_i \leq \sum_{j=1}^{m} \gamma_j x_{ij}, \quad \forall i$

Variational Inequality Formulation

Determine $(x^*, g^*, f^*) \in \mathbb{K}$ such that:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left[\frac{\partial c_{ij}^{a}(x_{ij}^*, f_i^*)}{\partial x_{ij}} + \frac{\partial c_{ij}^{t}(x_{ij}^*)}{\partial x_{ij}} + \frac{\partial c_{ij}^{m}(x_{ij}^*, g_i^*)}{\partial x_{ij}}\right] \times [x_{ij} - x_{ij}^*]$$

$$+ \sum_{i=1}^{n}\left[\frac{\partial c_i^{T}(g_i^*)}{\partial g_i} + \sum_{j=1}^{m}\frac{\partial c_{ij}^{m}(x_{ij}^*, g_i^*)}{\partial g_i}\right] \times [g_i - g_i^*]$$

$$+ \sum_{i=1}^{n}\left[\sum_{j=1}^{m}\frac{\partial c_{ij}^{a}(x_{ij}^*, f_i^*)}{\partial f_i}\right] \times [f_i - f_i^*] \geq 0, \quad \forall (x, g, f) \in \mathbb{K}$$

Variational Inequality Formulation

Assumptions

Let all the involved functions be continuously differentiable and strictly convex with respect to all variables.

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Standard Form

Find $X^* \in \mathcal{K} \subset \mathbb{R}^N$ such that $\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}$:

- $F^1_{ij}(X) \equiv \dfrac{\partial c^a_{ij}(x_{ij}, f_i)}{\partial x_{ij}} + \dfrac{\partial c^t_{ij}(x_{ij})}{\partial x_{ij}} + \dfrac{\partial c^m_{ij}(x_{ij}, g_i)}{\partial x_{ij}}$
- $F^2_i(X) \equiv \dfrac{\partial c^T_i(g_i)}{\partial g_i} + \displaystyle\sum_{j=1}^{m} \dfrac{\partial c^m_{ij}(x_{ij}, g_i)}{\partial g_i}$
- $F^3_i(X) \equiv \displaystyle\sum_{j=1}^{m} \dfrac{\partial c^a_{ij}(x_{ij}, f_i)}{\partial f_i}$

Existence Results

Theorem (Existence)

Let us assume that all the Assumptions are satisfied. Then, there exists at least one solution to the variational inequality

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Theorem (Strictly Monotonicity)

The function F defining the variational inequality is strictly monotone on $\mathcal{K}$, that is:

$$\langle (F(X^1) - F(X^2))^T, X^1 - X^2 \rangle > 0, \quad \forall X^1,\ X^2 \in \mathcal{K},\ X^1 \neq X^2$$

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Theorem (Uniqueness)

As F is strictly monotone in $\mathcal{K}$, then the variational inequality admits a unique solution

Lagrange Theory

Aim

To find an alternative formulation of the variational inequality governing the minimization problem for the optimal green area model by means of the Lagrange multipliers associated with the constraints defining the feasible set $\mathcal{K}$

Lagrange Theory

We set:

- $a_i = m_i - \sum_{j=1}^{m} x_{ij} - \sum_{j=1}^{m} \underline{x}_{ij} \leq 0, \quad \forall i$
- $b_i = \sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} \underline{x}_{ij} - u_i \leq 0, \quad \forall i$
- $q_i = \sum_{k=1}^{2} e_{ik} + e_{i3}(f_i) - \sum_{j=1}^{m} \alpha_j x_{ij} - \sum_{j=1}^{m} \alpha_j \underline{x}_{ij} \leq 0, \quad \forall i$
- $k_i = g_i - \sum_{j=1}^{m} \gamma_j x_{ij} \leq 0, \quad \forall i$
- $e_{ij} = -x_{ij} + \underline{x}_{ij} \leq 0, \quad n_i = -g_i \leq 0, \quad h_i = -f_i \leq 0, \quad \forall i, \ \forall j$
- $\Gamma(X) = (a_i, b_i, q_i, k_i, e_{ij}, n_i, h_i)_{i,\ j} \quad \mathcal{K} = \{X \in \mathbb{R}_{+}^{2nm+2n} : \Gamma(X) \leq 0\}$

Lagrange Theory

Lagrange Function

$$\begin{aligned}\mathcal{L}(X,\omega,\varphi,\vartheta,\lambda,\psi,\mu,\varepsilon) = \quad & V(x,g,f) + \sum_{i=1}^{n} \omega_i a_i \\ & + \sum_{i=1}^{n} \varphi_i b_i + \sum_{i=1}^{n} \vartheta_i q_i + \sum_{i=1}^{n} \lambda_i k_i \\ & + \sum_{i=1}^{n} \sum_{j=1}^{m} \psi_{ij} e_{ij} + \sum_{i=1}^{n} \mu_i n_i + \sum_{i=1}^{n} \varepsilon_i h_i,\end{aligned}$$

$$\forall X \in \mathbb{R}_+^{nm+2n},\ \forall \omega \in \mathbb{R}_+^n,\ \forall \varphi \in \mathbb{R}_+^n,\ \forall \vartheta \in \mathbb{R}_+^n,$$

$$\forall \lambda \in \mathbb{R}_+^n,\ \forall \psi \in \mathbb{R}_+^{nm},\ \forall \mu \in \mathbb{R}_+^n,\ \forall \varepsilon \in \mathbb{R}_+^n$$

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Equivalence

Variational Inequality Problem $\Longleftrightarrow \min_{x \in \mathcal{K}} V(x, g, f) = 0$

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Theorem

The minimization problem satisfies the Karush-Kuhn-Tucker conditions

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Equivalence

Variational Inequality Problem $\Longleftrightarrow$ $\min_{X\in\mathcal{K}} V(x,g,f)=0$

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The minimization problem satisfies the Karush-Kuhn-Tucker conditions

Theorem

Let X^* be the solution to the variational inequality, then the Lagrange multipliers, $\omega^*\in\mathbb{R}^n_+$, $\varphi^*\in\mathbb{R}^n_+$, $\vartheta^*\in\mathbb{R}^n_+$, $\lambda^*\in\mathbb{R}^n_+$, $\psi^*\in\mathbb{R}^{nm}_+$, $\mu^*\in\mathbb{R}^n_+$ and $\varepsilon^*\in\mathbb{R}^n_+$ associated with the constraints system do exist

Lagrange Theory

Saddle Point

$(X^*, \omega^*, \varphi^*, \vartheta^*, \lambda^*, \psi^*, \mu^*, \varepsilon^*)$ is a saddle point of the Lagrange function:

$$\begin{aligned} \mathcal{L}(X^*, \omega, \varphi, \vartheta, \lambda, \psi, \mu, \varepsilon, \lambda) \quad & \leq \mathcal{L}(X^*, \omega^*, \varphi^*, \vartheta^*, \lambda^*, \psi,^* \mu^*, \varepsilon^*) \\ & \leq \mathcal{L}(X, \omega^*, \varphi^*, \vartheta^*, \lambda^*, \psi,^* \mu^*, \varepsilon^*) \end{aligned}$$

$$\forall X \in \mathbb{R}_+^{nm+2n}, \ \forall \omega \in \mathbb{R}_+^n, \ \forall \varphi \in \mathbb{R}_+^n, \ \forall \vartheta \in \mathbb{R}_+^n,$$
$$\forall \lambda \in \mathbb{R}_+^n, \ \forall \psi \in \mathbb{R}_+^{nm}, \ \forall \mu \in \mathbb{R}_+^n, \ \forall \varepsilon \in \mathbb{R}_+^n$$

and

$$\begin{aligned} \omega_i^* a_i^* = 0, \ \varphi_i^* b_i^* = 0, \quad & \vartheta_i^* q_i^* = 0, \quad \forall i \\ \lambda_i^* h_i^* = 0, \ \mu_i^* n_i^* = 0, \quad & \varepsilon_i^* h_i^* = 0, \quad \forall i \\ & \psi_{ij}^* x_{ij}^* = 0, \quad \forall i, \ \forall j \end{aligned}$$

Additional conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_{ij}} &= \frac{\partial c_{ij}^a(x_{ij}^*, f_i^*)}{\partial x_{ij}} + \frac{\partial c_{ij}^t(x_{ij}^*)}{\partial x_{ij}} + \frac{\partial c_{ij}^m(x_{ij}^*, g_i^*)}{\partial x_{ij}} \\
& -\omega_i^* + \varphi_i^* - \alpha_j \vartheta_i^* - \gamma_j \lambda_i^* - \psi_{ij}^* = 0, \\
\frac{\partial \mathcal{L}}{\partial g_i} &= \frac{\partial c_i^T(g_i)}{\partial g_i} + \sum_{j=1}^{m} \frac{\partial c_{ij}^m(x_{ij}^*, g_i^*)}{\partial g_i} + \lambda_i^* - \mu_i^* = 0, \\
\frac{\partial \mathcal{L}}{\partial f_i} &= \sum_{j=1}^{m} \frac{\partial c_{ij}^a(x_{ij}^*, f_i^*)}{\partial f_i} + \vartheta_i^* \frac{\partial e_{i3}(f_i^*)}{\partial f_i} - \varepsilon_i^* = 0,
\end{aligned}
$$

Lagrange Theory

Interpretation of some Lagrange Multipliers

- If $a_i^* < 0$ and $b_i^* < 0 \iff m_i < \sum_{j=1}^{m} x_{ij}^* + \sum_{j=1}^{m} \underline{x}_{ij} < u_i \implies$
  $\omega_i^* = \varphi_i^* = 0$

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  $\omega_i^* = \varphi_i^* = 0$
- Also, if $x_{ij}^* > \underline{x}_{ij} \Longrightarrow \psi_{ij}^* = 0 \Longrightarrow$
  $\frac{\partial c_{ij}^a(x_{ij}^*, f_i^*)}{\partial x_{ij}} + \frac{\partial c_{ij}^t(x_{ij}^*)}{\partial x_{ij}} + \frac{\partial c_{ij}^m(x_{ij}^*, g_i^*)}{\partial x_{ij}} = \alpha_j \vartheta_i^* + \gamma_j \lambda_i^*$

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- If $\lambda_i^* > 0$ and $\vartheta_i^* > 0 \Longrightarrow \sum_{k=1}^{2} e_{ik} + e_{i3}(f_i^*) - \sum_{j=1}^{m} \alpha_j x_{ij}^* - \sum_{j=1}^{m} \alpha_j \underline{x}_{ij} = 0$

$$g_i^* - \sum_{j=1}^{m} \gamma_j x_{ij}^* = 0$$

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- CO_2 emissions in city i is completely absorbed by the optimal green area

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- CO_2 emissions in city i is completely absorbed by the optimal green area
- The number of employees reaches the maximum allowed number
- The total marginal cost associated with the expansion of green area increases
- Each additional unit of green area or employees is unnecessary
- Similar considerations hold even if only one between λ_i^* and ϑ_i^* is positive

Lagrange Theory

Interpretation of some Lagrange Multipliers

- If both $\lambda_i^* = 0$ and $\vartheta_i^* = 0 \Longrightarrow$
$$\sum_{k=1}^{2} e_{ik} + e_{i3}(f_i^*) - \sum_{j=1}^{m} \alpha_j x_{ij}^* - \sum_{j=1}^{m} \alpha_j \underline{x}_{ij} < 0 \text{ and } g_i^* - \sum_{j=1}^{m} \gamma_j x_{ij}^* < 0$$

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$$\sum_{k=1}^{2} e_{ik} + e_{i3}(f_i^*) - \sum_{j=1}^{m} \alpha_j x_{ij}^* - \sum_{j=1}^{m} \alpha_j \underline{x}_{ij} < 0 \text{ and } g_i^* - \sum_{j=1}^{m} \gamma_j x_{ij}^* < 0$$
- $\sum_{j=1}^{m} \left(c_{ij}^a(x_{ij}, f_i) + c_{ij}^t(x_{ij}) + c_{ij}^m(x_{ij}, g_i) \right)$ reaches its minimum value in x_{ij}^*

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- $\sum_{j=1}^{m} \left(c_{ij}^a(x_{ij}, f_i) + c_{ij}^t(x_{ij}) + c_{ij}^m(x_{ij}, g_i) \right)$ reaches its minimum value in x_{ij}^*
- If $\varphi_i^* > 0 \Longrightarrow \sum_{j=1}^{m} x_{ij}^* + \sum_{j=1}^{m} \underline{x}_{ij} = u_i$

Lagrange Theory

Interpretation of some Lagrange Multipliers

- If both $\lambda_i^* = 0$ and $\vartheta_i^* = 0 \Longrightarrow$
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- $\sum_{j=1}^{m} \left(c_{ij}^a(x_{ij}, f_i) + c_{ij}^t(x_{ij}) + c_{ij}^m(x_{ij}, g_i) \right)$ reaches its minimum value in x_{ij}^*
- If $\varphi_i^* > 0 \Longrightarrow \sum_{j=1}^{m} x_{ij}^* + \sum_{j=1}^{m} \underline{x}_{ij} = u_i$
- The optimal green area added to the pre-existing one reaches the maximum percentage of city i to be allocated to green areas.

Computational procedure

Euler Method and Convergence

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})) : \sum_{\tau=0}^{\infty} a_{\tau} = \infty,\ a_{\tau} > 0,\ a_{\tau} \to 0, \text{ as } \tau \to \infty$$

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Euler Method and Convergence

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Description

- **Step 0: Initialization** $X^0 \in \mathcal{K}$, $\tau = 1$
- **Step 1: Computation**
  $X^{\tau} \in \mathcal{K}$ solving the following variational inequality subproblem:

$$\langle X^{\tau} + a_{\tau} F(X^{\tau-1}) - X^{\tau-1}, X - X^{\tau} \rangle \geq 0, \quad \forall X \in \mathcal{K}$$

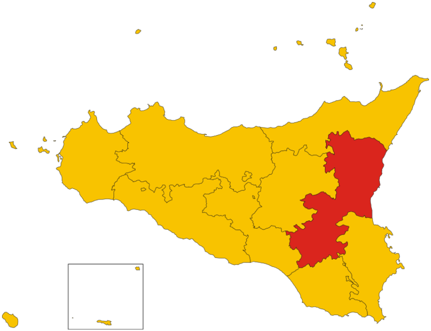
- **Step 2: Convergence**
  $\epsilon > 0$ and check whether $|X^{\tau} - X^{\tau+1}| \leq \epsilon$, then stop;
  otherwise,$\tau := \tau + 1$, and go to Step 1

Numerical Examples

Type of green area	Level of maintenance	Capacity of a km^2 to absorb CO_2: α_j
$j = 1$: Urban green area	High	$\alpha_1 = 545,000$ kg/yr
$j = 2$: Natural green area	Medium	$\alpha_2 = 569,070$ kg/yr

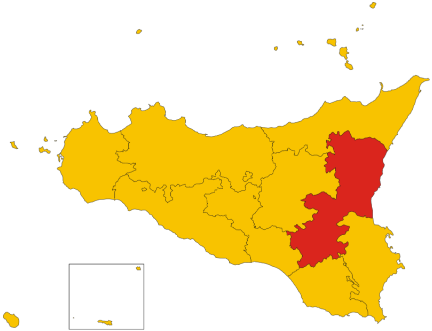
Table: Types of green areas considered in our model.

Metropolitan city of Catania



- 1,108,040 inhabitants

Metropolitan city of Catania



- 1,108,040 inhabitants
- 58 municipalities

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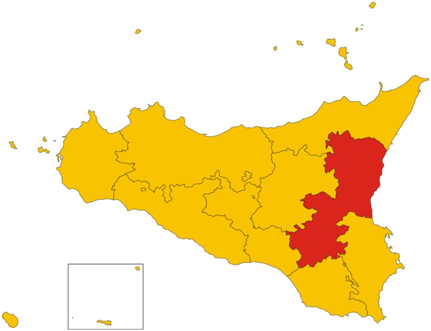

- 1,108,040 inhabitants
- 58 municipalities
- 3,570 km^2 extension

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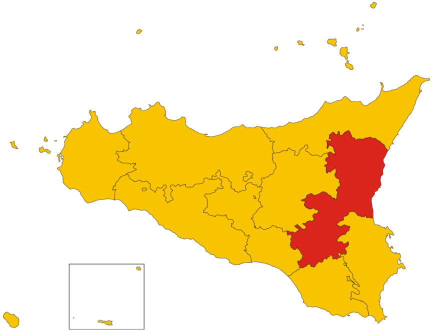

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- 3,570 km^2 extension
- 9 m^2 of green area available for every citizen

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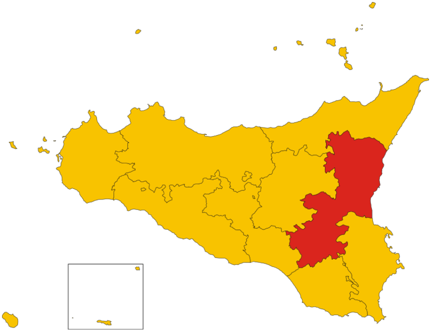

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- 3,570 km^2 extension
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- 9.97 km^2 of green area needed

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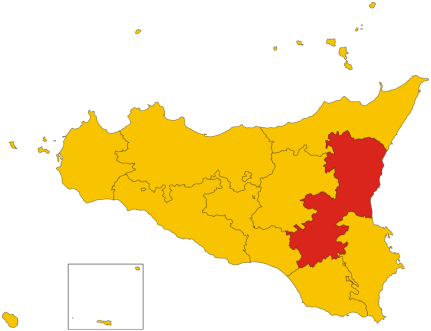


- 1,108,040 inhabitants
- 58 municipalities
- 3,570 km^2 extension
- 9 m^2 of green area available for every citizen
- 9.97 km^2 of green area needed
- 636.11 km^2 existing green area

Metropolitan city of Catania

Cost Functions

$$c^a_{11}(x_{11}, f_1) = 0.20x_{11}^2 - 0.15x_{11} + 1.20f_1^2 - 0.90f_1 + 13$$
$$c^a_{12}(x_{12}, f_1) = 0.30x_{12}^2 - 0.25x_{12} + 1.20f_1^2 - 0.90f_1 + 13$$
$$c^T_1(g_1) = 0.8g_1^2 - 0.6g_1 + 8 \; c^t_{11}(x_{11}) = 1.20x_{11}^2 - 1.10x_{11} + 8$$
$$c^t_{12}(x_{12}) = 1.20x_{12}^2 - 1.15x_{12} + 8$$
$$c^m_{11}(x_{11}, g_1) = 0.80x_{11}^2 - 0.70x_{11} + 0.8g_1^2 - 0.6g_1 + 27$$
$$c^m_{12}(x_{12}, g_1) = 0.50x_{12}^2 - 0.50x_{12} + 0.8g_1^2 - 0.6g_1 + 22$$

Metropolitan city of Catania

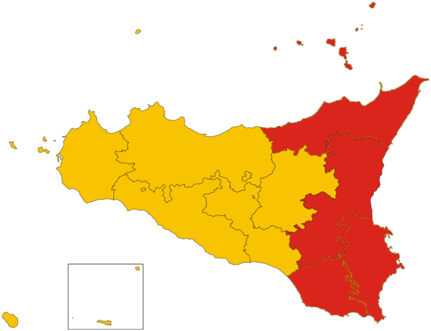
Cost Functions

$$c_{11}^{a}(x_{11}, f_1) = 0.20x_{11}^2 - 0.15x_{11} + 1.20f_1^2 - 0.90f_1 + 13$$
$$c_{12}^{a}(x_{12}, f_1) = 0.30x_{12}^2 - 0.25x_{12} + 1.20f_1^2 - 0.90f_1 + 13$$
$$c_1^T(g_1) = 0.8g_1^2 - 0.6g_1 + 8 \; c_{11}^{t}(x_{11}) = 1.20x_{11}^2 - 1.10x_{11} + 8$$
$$c_{12}^{t}(x_{12}) = 1.20x_{12}^2 - 1.15x_{12} + 8$$
$$c_{11}^{m}(x_{11}, g_1) = 0.80x_{11}^2 - 0.70x_{11} + 0.8g_1^2 - 0.6g_1 + 27$$
$$c_{12}^{m}(x_{12}, g_1) = 0.50x_{12}^2 - 0.50x_{12} + 0.8g_1^2 - 0.6g_1 + 22$$

Optimal Solution

$$x_{11}^* = 587.27 \text{ km}^2, \quad x_{12}^* = 703.47 \text{ km}^2,$$
$$g_1^* = 140,000, \quad f_1^* = 710,264$$

Eastern Sicily



- Catania
- Messina
- Syracuse
- Ragusa

Eastern Sicily

City	Surface (km^2)	Inhabithans	Emissions (kg/yr)	Pre-existing green area (km^2)
Catania	3570	1,108,040	e_{11}=637,635,890 e_{12}=728,210,000	$\underline{x}_{11}$=536 $\underline{x}_{12}$=100
Messina	3266.12	627,251	e_{21}=360,959,667 e_{22}=1,113,000,000	$\underline{x}_{21}$=48.99 $\underline{x}_{22}$=2.29
Syracuse	2124.13	397,341	e_{31}=228,654,421 e_{32}=1,020,000,000	$\underline{x}_{31}$=8.5 $\underline{x}_{32}$=120.9
Ragusa	1623.89	320,893	e_{41}=184,661,545 e_{42}=619,330,000	$\underline{x}_{41}$=6.5 $\underline{x}_{42}$=105.55

Eastern Sicily

Optimal Solution

$$x^*_{11} = 587.27 \text{ km}^2,\ x^*_{12} = 703.47 \text{ km}^2,\ g^*_1 = 140,000,\ f^*_1 = 710,264$$
$$x^*_{21} = 248.99 \text{ km}^2,\ x^*_{22} = 2.31 \text{ km}^2,\ g^*_2 = 257,600,\ f^*_2 = 341,283$$
$$x^*_{31} = 408.5 \text{ km}^2,\ x^*_{32} = 1536 \text{ km}^2,\ g^*_3 = 194,620,\ f^*_3 = 63,354$$
$$x^*_{41} = 93.5 \text{ km}^2,\ x^*_{42} = 225.98 \text{ km}^2,\ g^*_4 = 320,790,\ f^*_4 = 46,072$$

Metropolitan city of Catania

Remark

- It is necessary to increase the percentage of green area to counteract the CO_2 emissions

Metropolitan city of Catania

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- It is necessary to increase the percentage of green area to counteract the CO_2 emissions
- Syracuse: the new green area should be 91% of the total area of the city
- The optimal flow is less than the initial flow

Conclusions

- Optimization model for the management of green areas, Lagrange theory, computational procedure, and concrete examples

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- Optimization model for the management of green areas, Lagrange theory, computational procedure, and concrete examples
- The model could be further extended and improved, by introducing also budget constraints to the local organizations or increasing the awareness of inhabitants and of the industries with respect to environment and life, requiring, for instance, that every year a part of their revenue has to be destined to the improvement and maintenance of green areas